UNIVERSITY OF MICHIGAN

A Modern View of Scattering Amplitudes

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University of Michigan
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Yale Physics Colloquium

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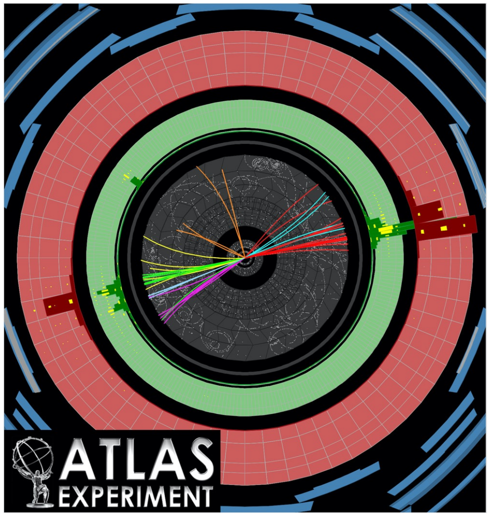
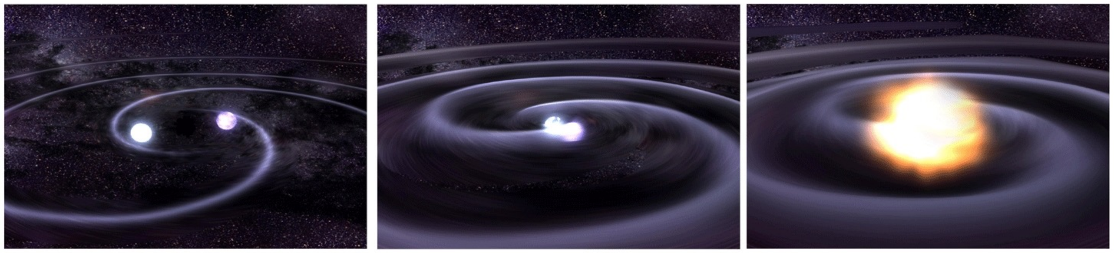
Particle Physics



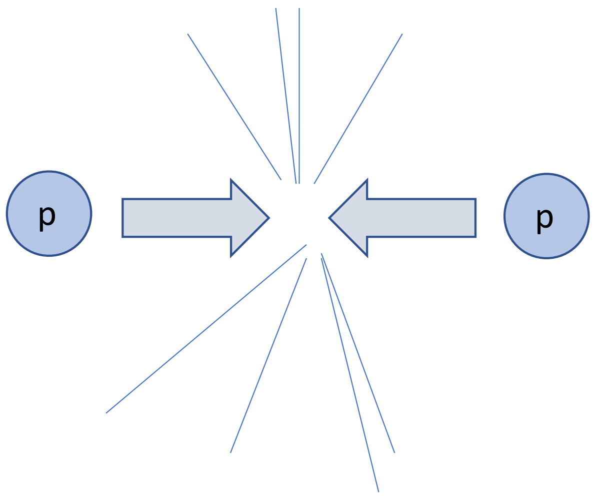
https://atlas.cern/updates/news/search-new-physics-processes-using-dijet-events

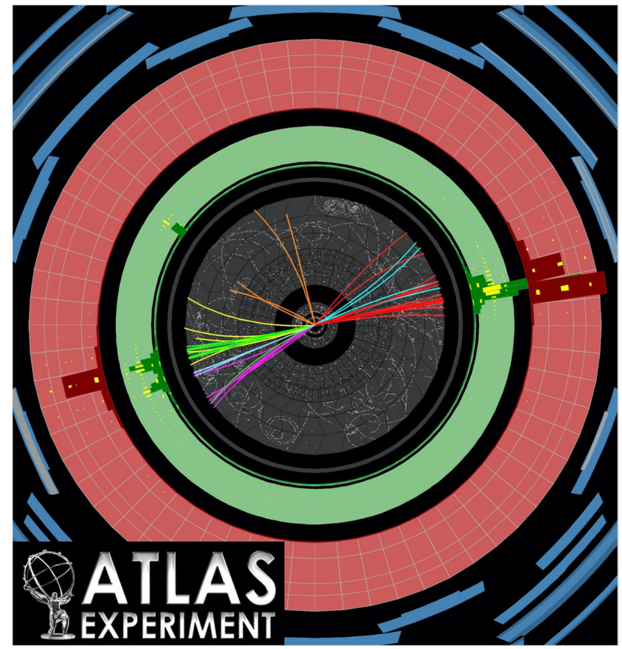
Gravity, gravitational waves, binary inspirals



Artist's impression [Image: NASA/CXC/GSFC/T.Strohmayer]

At the LHC at CERN, **protons** collide against **protons** to give rise to a spray of particles





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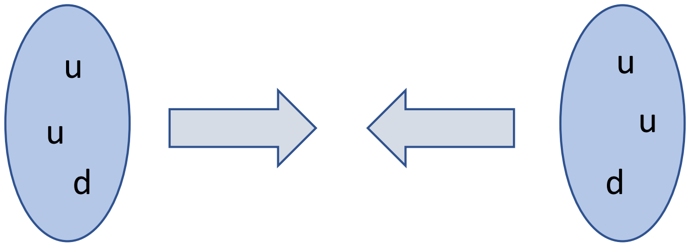
Zoom in… *partons* inside the protons



Quarks massive spin ½ fermions
up has charge +2/3
down has charge -1/3

Each proton has 2 up-quarks and 1 down-quark

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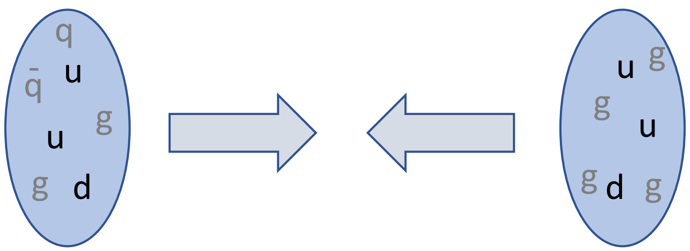

Each proton has 2 up-quarks and 1 down-quark

… and a quantum soup of quarks, anti-quarks, and gluons

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Gluons massless spin 1 boson, self-interacting

All this described by

QCD – Quantum Chromodynamics

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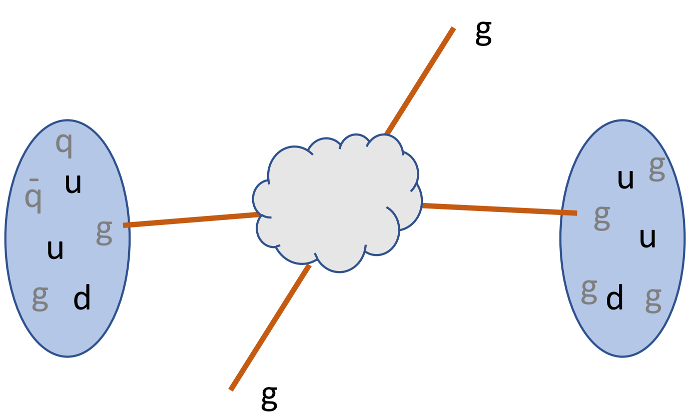

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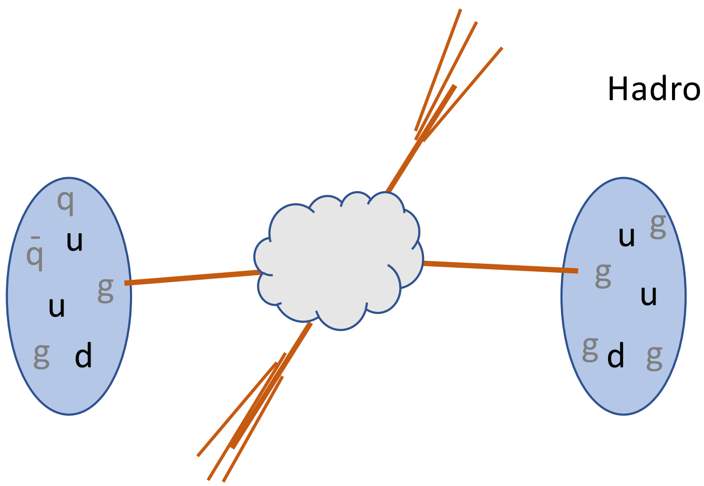

Hadronizes => jets

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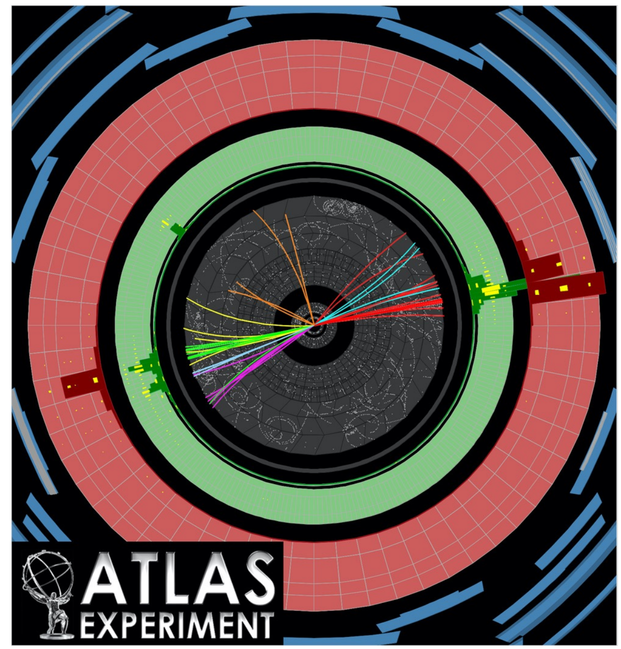
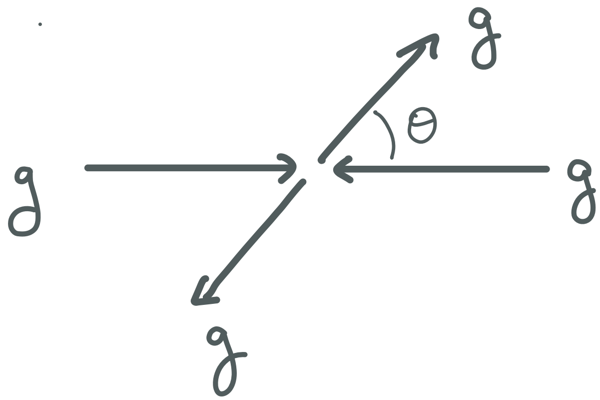


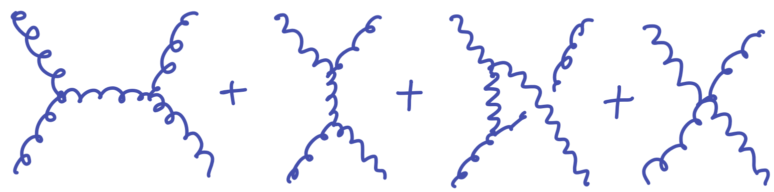


Differential cross-section: probability of process as function of energy-momentum and scattering angles.

$$\frac{d\sigma}{d\Omega} = \int |A_n|^2$$

Phase-space integral over the |**amplitude**|^2

The **scattering amplitude** can be computed as the sum over Feynman diagrams

$A_4(gg \to gg) =$ 

Perturbation theory in small coupling(s) => Tree-diagrams give the leading order contribution, loops are quantum corrections.

Gluon Amplitudes

$$\begin{array}{rcll} gg & \to & gg & 4 \text{ diagrams} \\ gg & \to & ggg & 25 \text{ diagrams} \\ gg & \to & gggg & 220 \text{ diagrams} \\ gg & \to & ggggg & 2485 \text{ diagrams} \\ gg & \to & gggggg & 34300 \text{ diagrams} \end{array}$$

[Mangano, Parke (1990)]

Feynman diagrams quickly become intractable… and this is only the leading order (tree-level)!

Yet, surprisingly, there exists very simple results for some of these amplitudes: *Gluons have helicity +1 or -1*

$$A_n[g^+g^+ \to g^+g^+ \dots g^+g^+] = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle \cdots \langle n1\rangle}$$

[Parke-Taylor formula (1986)]

Spinor-helicity formalism $\langle ij\rangle \sim \sqrt{(p_i.p_j)}$

Efficient Calculation of Scattering Amplitudes

Up through the 90's, a lot of new methods were developed to efficiently compute amplitudes relevant for particle phenomenology (high-energy parton scattering, W and Z production, Higgs,...) e.g.

- Berends-Giele off-shell recursion (1988+1990)
- String-inspired methods & Generalized Unitarity [Bern, Dixon, Kosower,…]

Including use of supersymmetry to organize the calculations. Espc. *N=4 Super Yang-Mills* became a fruitful "testing lab" for new methods.

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2003 Witten's twistor-string => conjectured new formalism for gluon amplitudes

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2003 Witten's twistor-string => conjectured new formalism for gluon amplitudes

$\Rightarrow$ on-shell recursion (BCFW)
$\Rightarrow$ CSW, RSV, …. Many interesting new amplitudes methods.

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Shock waves….?

2003…. I was in grad school busy with other research things:
Black Holes, Black Rings, and (later) Black Saturns,…

https://professional.ucsb.edu/international-programs

… so while I saw excitement about amplitudes igniting those around…

… I didn’t catch on…

.. not until early 2007…

Nov 2006

Is $\mathcal{N}=8$ Supergravity Ultraviolet Finite?

Z. Bern[a], L. J. Dixon[b], R. Roiban[c]
[a] *Department of Physics and Astronomy, UCLA, Los Angeles, CA 90095-1547, USA*
[b] *Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA*
[c] *Department of Physics, Pennsylvania State University, University Park, PA 16802, USA*

Conventional wisdom holds that no four-dimensional gravity field theory can be ultraviolet finite. This understanding is based mainly on power counting. Recent studies confirm that one-loop $\mathcal{N}=8$ supergravity amplitudes satisfy the so-called "no-triangle hypothesis", which states that triangle and bubble integrals cancel from these amplitudes. A consequence of this hypothesis is that for any number of external legs, at one loop $\mathcal{N}=8$ supergravity and $\mathcal{N}=4$ super-Yang-Mills have identical superficial degrees of ultraviolet behavior in D dimensions. We describe how the unitarity method allows us to promote these one-loop cancellations to higher loops, suggesting that previous power counts were too conservative. We discuss higher-loop evidence suggesting that $\mathcal{N}=8$ supergravity has the same degree of divergence as $\mathcal{N}=4$ super-Yang-Mills theory and is ultraviolet finite in four dimensions. We comment on calculations needed to reinforce this proposal, which are feasible using the unitarity method.

Conventional story:
Loops are UV divergent, need renormalization.
Gravity non-renormalizable
$\Rightarrow$ Problem unifying QM & GR

But maybe with enough *supersymmetry*: could it be that all loop-divergencies cancel???

4-graviton amplitude shown to be UV finite up to an incl. 3-loops

4-loops? Higher point?

My interest: what do the symmetries have to say about these finiteness results?

What are Gravity Amplitudes?

Just like E&M is a force mediated by the **photon**, and QCD has **gluons** Massless spin 1

gravity has **gravitons** Massless spin 2

What do we mean by "gravity"? Einstein General Relativity => Einstein's equation

EOM of the Einstein-Hilbert action $$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R$$ spacetime curvature

G is Newton's constant. In units with $c = \hbar = 1$ $\quad M_{\mathrm{Planck}} = G^{-1/2} \approx 10^{19}\,\mathrm{GeV}$

Perturbative Gravity

Flat space (no curvature)

fluctuation

Expand the metric in fluctuations around flat space $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ $\kappa = \sqrt{8\pi G}$

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R \;\; = \int d^4x\left(h\partial^2 h + \ldots\right)$$ *schematically*

Wave equation fluctuations of spacetime: **gravitational waves!!!**

Famously detected by LIGO: first event on Sep 14, 2015, announced Feb 2016

2017 Nobel Prize (Weiss, Barish, Thorne)

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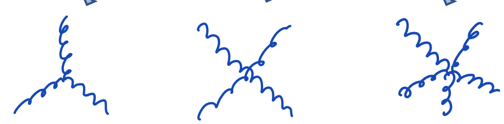
$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R \;=\int d^4x\left(h\partial^2 h + \kappa\partial^2 h^3 + \kappa^2\partial^2 h^4 + \kappa^3\partial^2 h^5 + \dots\right)$$ *schematically*

propagator



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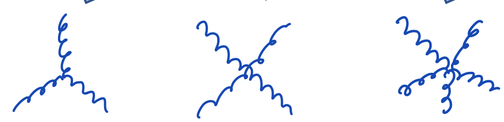
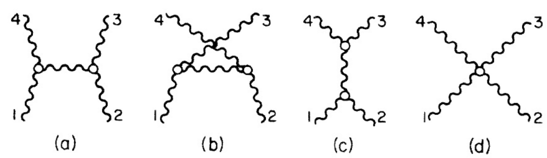


The graviton vertex Feynman rules are complicated…

4-point amplitude:



[Sannan (1986)]

$$\begin{aligned}(\mathrm{a})_{\text{gravity}} = i\kappa^2\Bigg[&\left[\left(\frac{t}{2}+\frac{su}{4t}\right](e_1e_4)(e_2e_3)-\frac{t}{2}[(e_1e_4e_2e_3)+(e_1e_2e_3e_4)]\\
&+[(k_2e_3k_2)(e_1e_2e_4)+(k_3e_2k_3)(e_1e_3e_4)+(k_1e_4k_1)(e_1e_2e_3)+(k_4e_1k_4)(e_2e_3e_4)]\\
&+\tfrac{1}{2}(e_1e_4)[(k_1e_2e_3k_4)+(k_4e_2e_3k_1)+2(k_1e_2e_3k_1)+2(k_4e_2e_3k_4)+3(k_3e_2e_3k_2)]\\
&+\tfrac{1}{2}(e_2e_3)[(k_2e_1e_4k_3)+(k_3e_1e_4k_2)+2(k_2e_1e_4k_2)+2(k_3e_1e_4k_3)+3(k_4e_1e_4k_1)]\\
&-\frac{u}{2t}(e_1e_4)[(k_1e_2e_3k_2)+(k_3e_2e_3k_4)+2(k_3e_2e_3k_1)+2(k_4e_2e_3k_2)]\\
&-\frac{s}{2t}(e_1e_4)[(k_3e_2e_3k_1)+(k_4e_2e_3k_2)+2(k_1e_2e_3k_2)+2(k_3e_2e_3k_4)]\\
&-\frac{u}{2t}(e_2e_3)[(k_1e_4e_1k_2)+(k_3e_4e_1k_4)+2(k_1e_4e_1k_3)+2(k_2e_4e_1k_4)]\\
&-\frac{s}{2t}(e_2e_3)[(k_1e_4e_1k_3)+(k_2e_4e_1k_4)+2(k_1e_4e_1k_2)+2(k_3e_4e_1k_4)]\end{aligned}$$

+ about 3 times as many terms

Do we care about graviton scattering?

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R \;= \int d^4x\left(h\partial^2 h + \kappa\partial^2 h^3 + \kappa^2\partial^2 h^4 + \kappa^3\partial^2 h^5 + \ldots\right)$$ *schematically*

The graviton interactions are controlled by $\kappa = \sqrt{8\pi G}$ which is dimensionful.

Proper dimensionless coupling is terms of the characteristic energy *E* of the process $E\kappa$

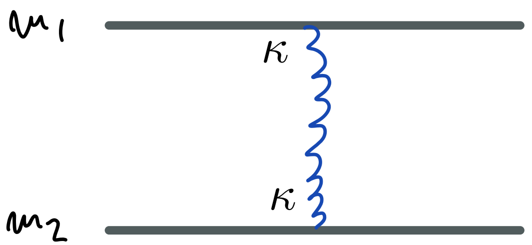
But with $\kappa \sim 10^{-19}\,\mathrm{GeV}^{-1}$, even for LHC energies E ~ 10 TeV ~ 10^4 GeV, the effective coupling is very very small.

So why care about perturbative gravity and graviton self-interactions????

Tree level scattering captures the classical physics, i.e. same physics as Einstein's equations.

Do we care about graviton scattering? Yes!

Equivalence principle: gravitons interact with all other matter with the same coupling $\kappa = \sqrt{8\pi G}$



$$V = -\frac{G m_1 m_2}{r}$$

Two classes of corrections to this:

1) Moving masses => non-relativistic expansion in small *v/c* — Post-Newtonian

2) Higher powers in G — Post-Minkowskian

The effective description of Black hole binary inspirals have

$$v^2 \sim \frac{Gm}{r} \ll 1$$

(Virial Theorem)

Do we care about graviton scattering? Yes!

Binary black hole inspirals:

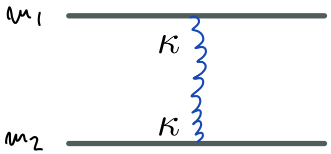
Corrections in G as important as corrections in relative velocity v

$$
\begin{array}{lccccccccccccccccccc}
 & \mathbf{0PN} & & \mathbf{1PN} & & \mathbf{2PN} & & \mathbf{3PN} & & \mathbf{4PN} & & \mathbf{5PN} & & \mathbf{6PN} & & \mathbf{7PN} & & & \\
\mathbf{1PM} & (\,1 & + & v^2 & + & v^4 & + & v^6 & + & v^8 & + & v^{10} & + & v^{12} & + & v^{14} & + & \ldots) & G^1 \\
\mathbf{2PM} & & & (\,1 & + & v^2 & + & v^4 & + & v^6 & + & v^8 & + & v^{10} & + & v^{12} & + & \ldots) & G^2 \\
\mathbf{3PM} & & & & & (\,1 & + & v^2 & + & v^4 & + & v^6 & + & v^8 & + & v^{10} & + & \ldots) & G^3 \\
\mathbf{4PM} & & & & & & & (\,1 & + & v^2 & + & v^4 & + & v^6 & + & v^8 & + & \ldots) & G^4 \\
\mathbf{5PM} & & & & & & & & & (\,1 & + & v^2 & + & v^4 & + & v^6 & + & \ldots) & G^5 \\
\mathbf{6PM} & & & & & & & & & & & (\,1 & + & v^2 & + & v^4 & + & \ldots) & G^6 \\
 & & & & & & & & & & & & \vdots & & & & & &
\end{array}
$$

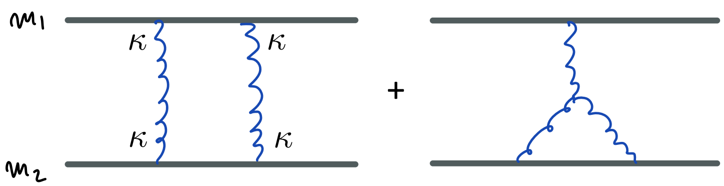
[from: Bern, Cheung, Roiban, Shen, Solon (2019)]

Also: Gravitational EFT for extended objects
[Goldberger + Rothstein 2004]

Higher powers in G?



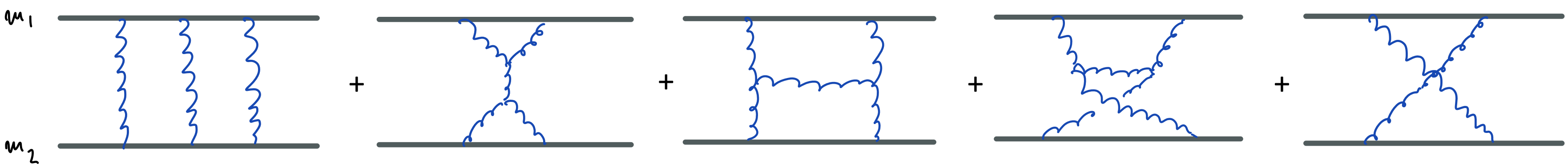
$$V = -\frac{G m_1 m_2}{r}$$



+ ...

$$O(G^2)$$

[Bjerrum-Bohr, Donoghue, Holstein (2002)]



+ ...

$$O(G^3)$$

Bern, Cheung, Roiban, Shen, Solon (2019)]

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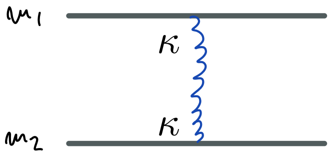
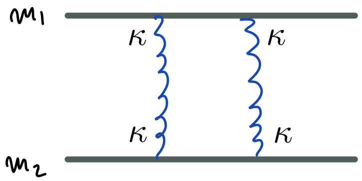


$$V = -\frac{G m_1 m_2}{r}$$



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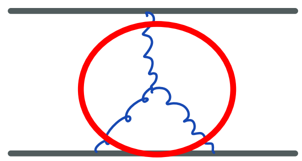
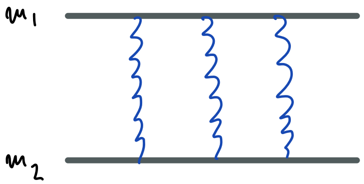


+ ... $O(G^2)$ [Bjerrum-Bohr, Donoghue, Holstein (2002)]



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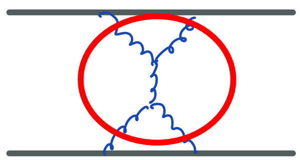

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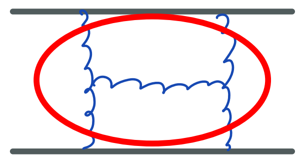

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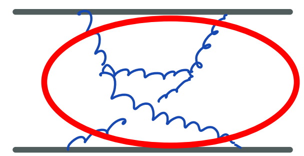

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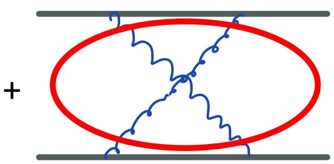


+ ... $O(G^3)$

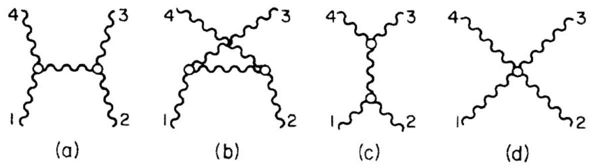
Graviton amplitudes

Bern, Cheung, Roiban, Shen, Solon (2019)]

So, graviton scattering *is* something we care about!

(but is sounded like it was very complicated... with Feynman rules anyway)

4-point amplitude:



[Sannan (1986)]

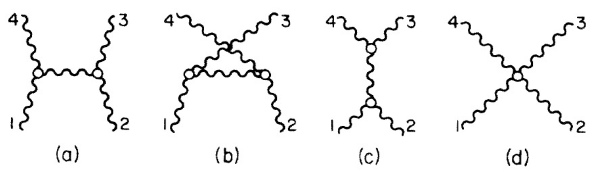
$$
\begin{aligned}
(\mathrm{a})_{\text{gravity}} = \; & i\kappa^2 \Bigg[\left[\left(\frac{t}{2} + \frac{su}{4t} \right] (e_1 e_4)(e_2 e_3) - \frac{t}{2}[(e_1 e_4 e_2 e_3) + (e_1 e_2 e_3 e_4)] \right. \\
& + [(k_2 e_3 k_2)(e_1 e_2 e_4) + (k_3 e_2 k_3)(e_1 e_3 e_4) + (k_1 e_4 k_1)(e_1 e_2 e_3) + (k_4 e_1 k_4)(e_2 e_3 e_4)] \\
& + \tfrac{1}{2}(e_1 e_4)[(k_1 e_2 e_3 k_4) + (k_4 e_2 e_3 k_1) + 2(k_1 e_2 e_3 k_1) + 2(k_4 e_2 e_3 k_4) + 3(k_3 e_2 e_3 k_2)] \\
& + \tfrac{1}{2}(e_2 e_3)[(k_2 e_1 e_4 k_3) + (k_3 e_1 e_4 k_2) + 2(k_2 e_1 e_4 k_2) + 2(k_3 e_1 e_4 k_3) + 3(k_4 e_1 e_4 k_1)] \\
& - \frac{u}{2t}(e_1 e_4)[(k_1 e_2 e_3 k_2) + (k_3 e_2 e_3 k_4) + 2(k_3 e_2 e_3 k_1) + 2(k_4 e_2 e_3 k_2)] \\
& - \frac{s}{2t}(e_1 e_4)[(k_3 e_2 e_3 k_1) + (k_4 e_2 e_3 k_2) + 2(k_1 e_2 e_3 k_2) + 2(k_3 e_2 e_3 k_4)] \\
& - \frac{u}{2t}(e_2 e_3)[(k_1 e_4 e_1 k_2) + (k_3 e_4 e_1 k_4) + 2(k_1 e_4 e_1 k_3) + 2(k_2 e_4 e_1 k_4)] \\
& - \frac{s}{2t}(e_2 e_3)[(k_1 e_4 e_1 k_3) + (k_2 e_4 e_1 k_4) + 2(k_1 e_4 e_1 k_2) + 2(k_3 e_4 e_1 k_4)] \\
& - \frac{1}{t}(e_1 e_4)[(k_1 e_2 k_1)(k_2 e_3 k_2) + (k_3 e_2 k_3)(k_1 e_3 k_1) + (k_4 e_2 k_1)(k_2 e_3 k_2) \quad + \ldots
\end{aligned}
$$

Four such considerably complicated expressions from Feynman diagrams. Add them up, and…

So, graviton scattering *is* something we care about!

(but is sounded like it was very complicated... with Feynman rules anyway)

4-point amplitude:



[Sannan (1986)]

$$
\begin{aligned}
(\mathrm{a})_{\text{gravity}} = \; & i\kappa^2\Big[\Big(\frac{t}{2}+\frac{su}{4t}\Big)(e_1e_4)(e_2e_3)-\frac{t}{2}[(e_1e_4e_2e_3)+(e_1e_2e_3e_4)] \\
& +[(k_2e_3k_2)(e_1e_2e_4)+(k_3e_2k_3)(e_1e_3e_4)+(k_1e_4k_1)(e_1e_2e_3)+(k_4e_1k_4)(e_2e_3e_4)] \\
& +\tfrac{1}{2}(e_1e_4)[(k_1e_2e_3k_4)+(k_4e_2e_3k_1)+2(k_1e_2e_3k_1)+2(k_4e_2e_3k_4)+3(k_3e_2e_3k_2)] \\
& +\tfrac{1}{2}(e_2e_3)[(k_2e_1e_4k_3)+(k_3e_1e_4k_2)+2(k_2e_1e_4k_2)+2(k_3e_1e_4k_3)+3(k_4e_1e_4k_1)] \\
& -\frac{u}{2t}(e_1e_4)[(k_1e_2e_3k_2)+(k_3e_2e_3k_4)+2(k_3e_2e_3k_1)+2(k_4e_2e_3k_2)] \\
& -\frac{s}{2t}(e_1e_4)[(k_3e_2e_3k_1)+(k_4e_2e_3k_2)+2(k_1e_2e_3k_2)+2(k_3e_2e_3k_4)] \\
& -\frac{u}{2t}(e_2e_3)[(k_1e_4e_1k_2)+(k_3e_4e_1k_4)+2(k_1e_4e_1k_3)+2(k_2e_4e_1k_4)] \\
& -\frac{s}{2t}(e_2e_3)[(k_1e_4e_1k_3)+(k_2e_4e_1k_4)+2(k_1e_4e_1k_2)+2(k_3e_4e_1k_4)] \\
& -\frac{1}{t}(e_1e_4)[(k_1e_2k_1)(k_2e_3k_2)+(k_3e_2k_3)(k_1e_3k_1)+(k_4e_2k_1)(k_2e_3k_2) \quad + \dots
\end{aligned}
$$

Four such considerably complicated expressions from Feynman diagrams. Add them up, and…

… rather amazingly that the sum is simplifies to

$$A_4^{\text{gravity}} = s\, A_4^{\text{YM}}[1234]\, A_4^{\text{YM}}[1243]$$

$hh \to hh$ $\quad (p_1+p_2)^2 \quad gg \to gg \quad gg \to gg$

Double-Copy

Similar relations for higher-point tree amplitudes too!

Double-Copy

gravity = (Yang-Mills $)^2$

Originally from string theory
[Kawai-Lewellen-Tye (1986)]
KLT relations

Many applications:

Explore the UV structure of supergravity theories (finiteness?)

Gravitational radiation (3PM)

Classical double-copy (EOM)

Enhancement of symmetries

Properties of string amplitudes

Generalizations to (A)dS

Chiral perturbation theory (e.g. pions) -> galileons

Side note

(before I tell you more about the double-copy)

A few points in Amplitudes History

1960s S-matrix program
1986 KLT double-copy relations
mid-1980s Develop methods for amplitudes (off-shell recursion, generalized unitarity, …)
2003 Witten's twistor string missed that bus…
2006+07 Is 4d N=8 supergravity UV finite? 3-loop finiteness at 4-pt caught on =>

2007 my first paper on amplitudes was about a new double-copy tree-level formula.
[HE, Freedman (2007)]

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2018	5-loop finiteness at 4-pt [Bern, Carrasco, Chen, Edison, Johanson (2018)]
2021	**The Double-Copy Bootstrap** [Chi, HE, Herderschee, Jones, Paranjape (2021)]
…	

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That's what I'll tell you about next

Double-Copy

gravity = (Yang-Mills)^2

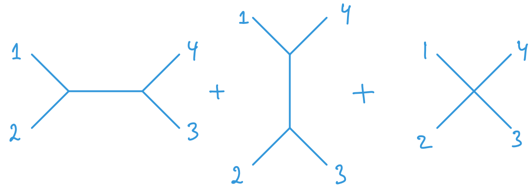
How can the double-copy possibly work?????

YM gluon amplitudes can be color-ordered: $A_n(1^{a_1}2^{a_2}\ldots n^{a_n}) = \sum A_n[12\ldots n]\mathrm{Tr}(T^{a_1}T^{a_2}\cdots T^{a_n})$

kinematics

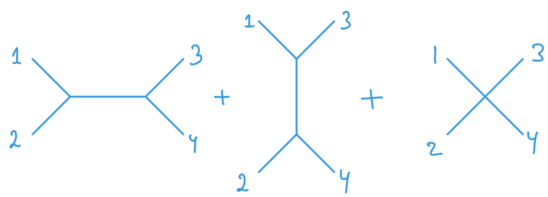
group theory

$A_4[1234]$ =



Poles $\quad \frac{1}{s} \quad \frac{1}{u}$

$A_4[1243]$ =



$\frac{1}{s} \quad \frac{1}{t}$

So that means

$s = (p_1 + p_2)^2$

$t = (p_1 + p_3)^2$

$u = (p_1 + p_4)^2$

$A_4[1234]\, A_4[1243]$

has a double-pole $\frac{1}{s^2}$

No good, because the graviton ampl has simple poles in s,t and u

What comes to the rescue is the multiplication **KERNEL**

$$M_4 = -s A_4[1234] A_4[1243]$$

That's what makes the pole-structure work out!!

Similarly for another form of the double-copy:

$$M_4 = -\frac{su}{t} A_4[1234] A_4[1234]$$

These two relations are examples **of field theory KLT formulas** at 4-point [Kawai-Lewellen-Tye (1985)]

The double-copy kernel:

1) Eliminates double-poles from $A_4 * A_4$
2) Provides "missing" poles

Another important aspect of field theory KLT: **KKBCJ relations**

$$M_4 = -s A_4[1234] A_4[1243] \qquad \textit{and} \qquad M_4 = -\frac{su}{t} A_4[1234] A_4[1234]$$

Another important aspect of field theory KLT: KKBCJ relations

$$M_4 = -s A_4[1234] A_4[1243] \qquad \textit{and} \qquad M_4 = -\frac{su}{t} A_4[1234] A_4[1234]$$

Their difference must be zero, i.e.

$$0 = A_4[1243] - \frac{u}{t} A_4[1234]$$

And this **is** true for YM amplitudes.

This is an example of a **BCJ** (Bern-Carrasco-Johansson) **relation** at 4-point.

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$$\left.\begin{array}{l}\text{Kleiss-Kuijf}\left\{\begin{array}{rl}\text{Trace-reversal:} & \mathcal{A}_4[1432] = \mathcal{A}_4[1234]\,, \quad \textit{etc} \\ U(1)\text{-decoupling:} & \mathcal{A}_4[1234] + \mathcal{A}_4[1243] + \mathcal{A}_4[1423] = 0\,,\end{array}\right. \\ \qquad\qquad\qquad\qquad \text{BCJ:} \quad \mathcal{A}_4[1234] - \dfrac{t}{u}\mathcal{A}_4[1243] = 0\,.\end{array}\right\} \text{“KKBCJ relations”}$$

Upshot

e.g. gluon amplitudes in Yang-Mills theory

- There exists **a double-copy** procedure that maps products of color-ordered tree-amplitudes in two theories into amplitudes of another theory.

e.g. graviton amplitudes in General Relativity (+ dilaton & axion)

Upshot

e.g. gluon amplitudes in Yang-Mills theory

- There exists **a double-copy** procedure that maps products of color-ordered tree-amplitudes in two theories into amplitudes of another theory.

e.g. graviton amplitudes in General Relativity (+ dilaton & axion)

- Not all theories can be double-copied: must have tree-amplitudes that satisfy the **KKBCJ relations**

examples

YM theory ✓ Chiral perturbation theory (pions) ✓

Super YM theory ✓ Bi-adjoint scalar model ✓

The double-copy is a **map** FT x FT -> FT

The kernel defines the **multiplication rule** of this map

Multiplication table for FT x FT -> FT

FT $\otimes$ FT	YM	$\mathcal{N}=4$ SYM	χPT	BAS
YM	gravity+	$\mathcal{N}=4$ SG	BI	YM
$\mathcal{N}=4$ SYM	$\mathcal{N}=4$ SG	$\mathcal{N}=8$ SG	$\mathcal{N}=4$ sDBI	$\mathcal{N}=4$ SYM
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SG = Supergravity = gravity + supersymmetry

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BI = Born-Infeld nonlinear electrodynamics [Born and Infeld (1937)]

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N=4 sDBI = N=4 supersymmetric Dirac-Born-Infeld theory = low-energy effective action on D3-branes in string theory

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BAS	YM	$\mathcal{N}=4$ SYM	χPT	BAS

sGalileon = special Galileon (shows up in cosmology, effective brane actions, massive gravity, various model building)

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Cubic Bi-Adjoint Scalar model (BAS)

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This map has an ***identity element*** **1**: the **bi-adjoint scalar model** (BAS)

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KLT algebra

$$\mathrm{L} = \mathrm{L} \otimes \mathbf{1}\,, \qquad \mathrm{R} = \mathbf{1} \otimes \mathrm{R}\,, \qquad \mathbf{1} = \mathbf{1} \otimes \mathbf{1}\,.$$

My recent work

Understand the double-copy structure better

$$A_{L\otimes R} = A_L \otimes A_R$$

$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S_n[a|b]\, A_n^{\mathrm{R}}[b]$$

double copy kernel

The multiplication rule is defined by the double-copy kernel and it has an identity element

$$\mathrm{L} = \mathrm{L} \otimes \mathbf{1}\,, \quad \mathrm{R} = \mathbf{1} \otimes \mathrm{R}\,, \quad \mathbf{1} = \mathbf{1} \otimes \mathbf{1}\,.$$

When the multiplication rule is changed,
the identity element is changed, and vice versa:
The kernel and the identity model are uniquely linked!

Double-copy bootstrap

$$\mathrm{L} = \mathrm{L} \otimes \mathbf{1}\,, \quad \mathrm{R} = \mathbf{1} \otimes \mathrm{R}\,, \quad \underbrace{\mathbf{1} = \mathbf{1} \otimes \mathbf{1}}_{\text{KLT Bootstrap Equation}}\,.$$

Modify the BAS model <=> new kernel

Double-copy bootstrap

$$\underbrace{\mathrm{L} = \mathrm{L} \otimes \mathbf{1}\,, \quad \mathrm{R} = \mathbf{1} \otimes \mathrm{R}\,,}_{\text{Generalize the KKBCJ / monodromy relations}} \quad \underbrace{\mathbf{1} = \mathbf{1} \otimes \mathbf{1}\,.}_{\text{KLT Bootstrap Equation}}$$

Generalize the **KKBCJ / monodromy** relations

KLT Bootstrap Equation

We construct new double-kernels, specifically aimed at double-copies of Effective Field Theories (EFTs)

Gives a new formalism for understanding the structure of the double-copy.

Opens a new door for studies of the effective theories and their double-copy relationships

Generalizations of the Double-Copy: the KLT Bootstrap

Huan-Hang Chi[a] **Henriette Elvang**[a] **Aidan Herderschee**[a] **Callum R. T. Jones**[a,b] **Shruti Paranjape**[a]

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The University of Michigan, Ann Arbor, MI 48109-1040, USA

[b] *Mani L. Bhaumik Institute for Theoretical Physics, Department of Physics and Astronomy, UCLA, Los Angeles, CA 90095, USA*

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Abstract: We formulate a new program to generalize the double-copy of tree amplitudes. The approach exploits the link between the identity element of t… algebra" and the KLT kernel and we demonstrate how this leads t… …ap equations that the double-copy kernel has to satisf… …constraints). We solve the KLT bootstrap equation… …higher-derivative corrections to the 4-… …kernel generalizes the string KL… …ns. It admits new color-structures in the… …it provides distinct generalized KK and BCJ relatio… …single-color theories and is in that sense a 'heterotic'-type double-… …ustrate the generalized double-copy in detail for 4d Yang-Mills theory with higher-derivative corrections that produce dilaton-axion-gravity with local operators up order $\nabla^{10}R^4$. Finally, we initiate a search for new double-copies kernels.

Use the KLT algebra as framework for new more general double-copies

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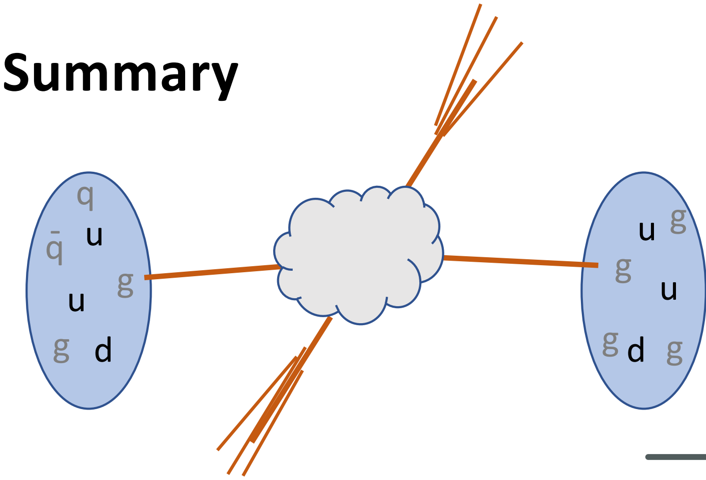
Postdoc at
UC Davis

Aidan Herderschee
4th year graduate student

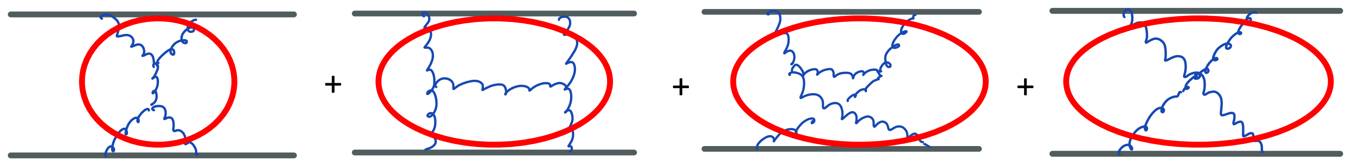
Callum Jones
PhD 2020
Postdoc at UCLA

Summary



High-energy gluon scattering at particle colliders

Post-Minkowskian corrections to binary inspiral effective Hamiltonian



gravity = (Yang-Mills)^2

New bootstrap formalism for the double-copy

$$\mathrm{L} = \mathrm{L} \otimes \mathbf{1}\,, \qquad \mathrm{R} = \mathbf{1} \otimes \mathrm{R}\,, \qquad \mathbf{1} = \mathbf{1} \otimes \mathbf{1}\,.$$

Landscape

same KLT kernel

$$A_n^{\mathrm{L}\otimes\mathrm{R}} = \sum_{a,b} A_n^{\mathrm{L}}[a]\, S[a|b]\, A_n^{\mathrm{R}}[b]$$

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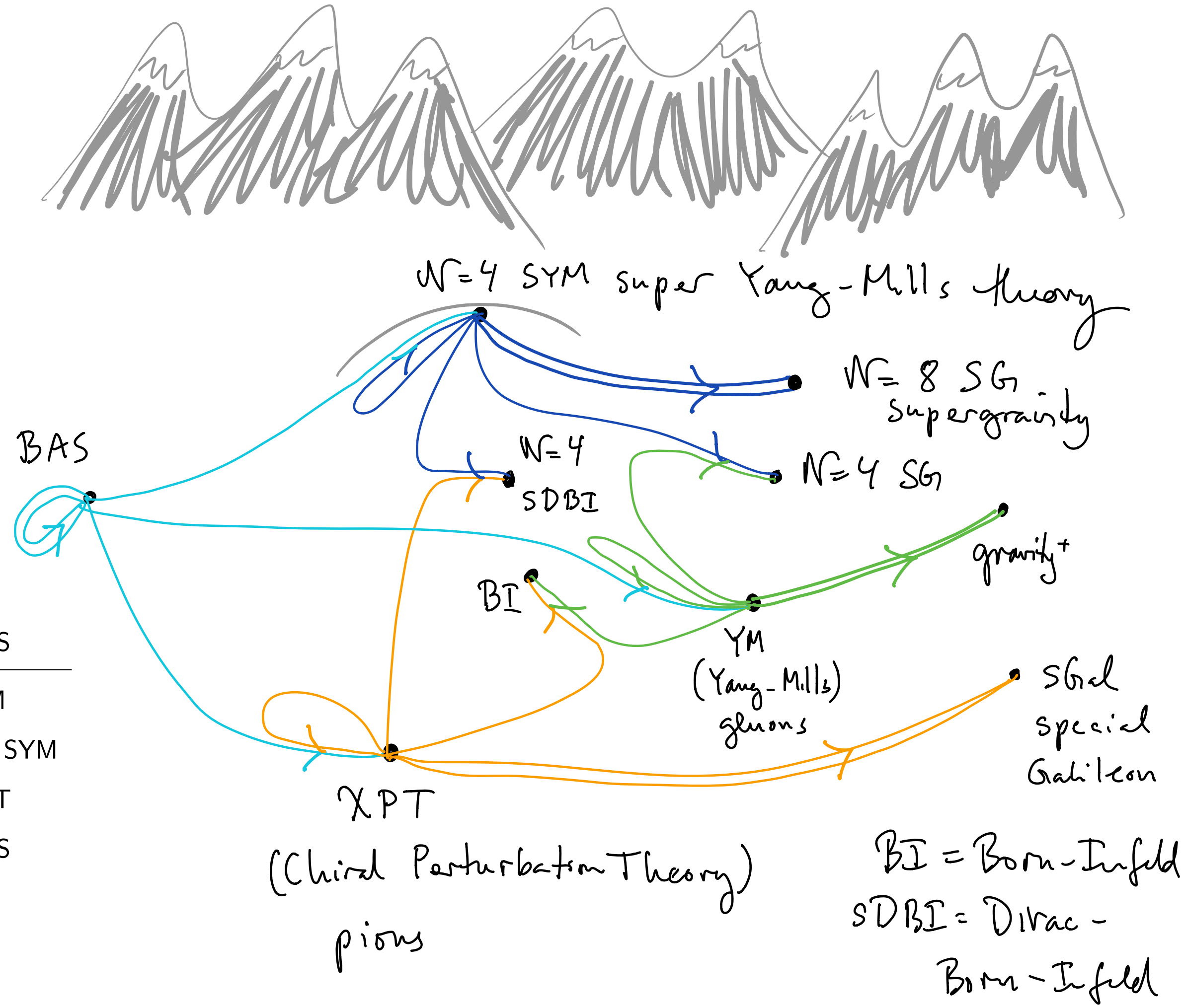

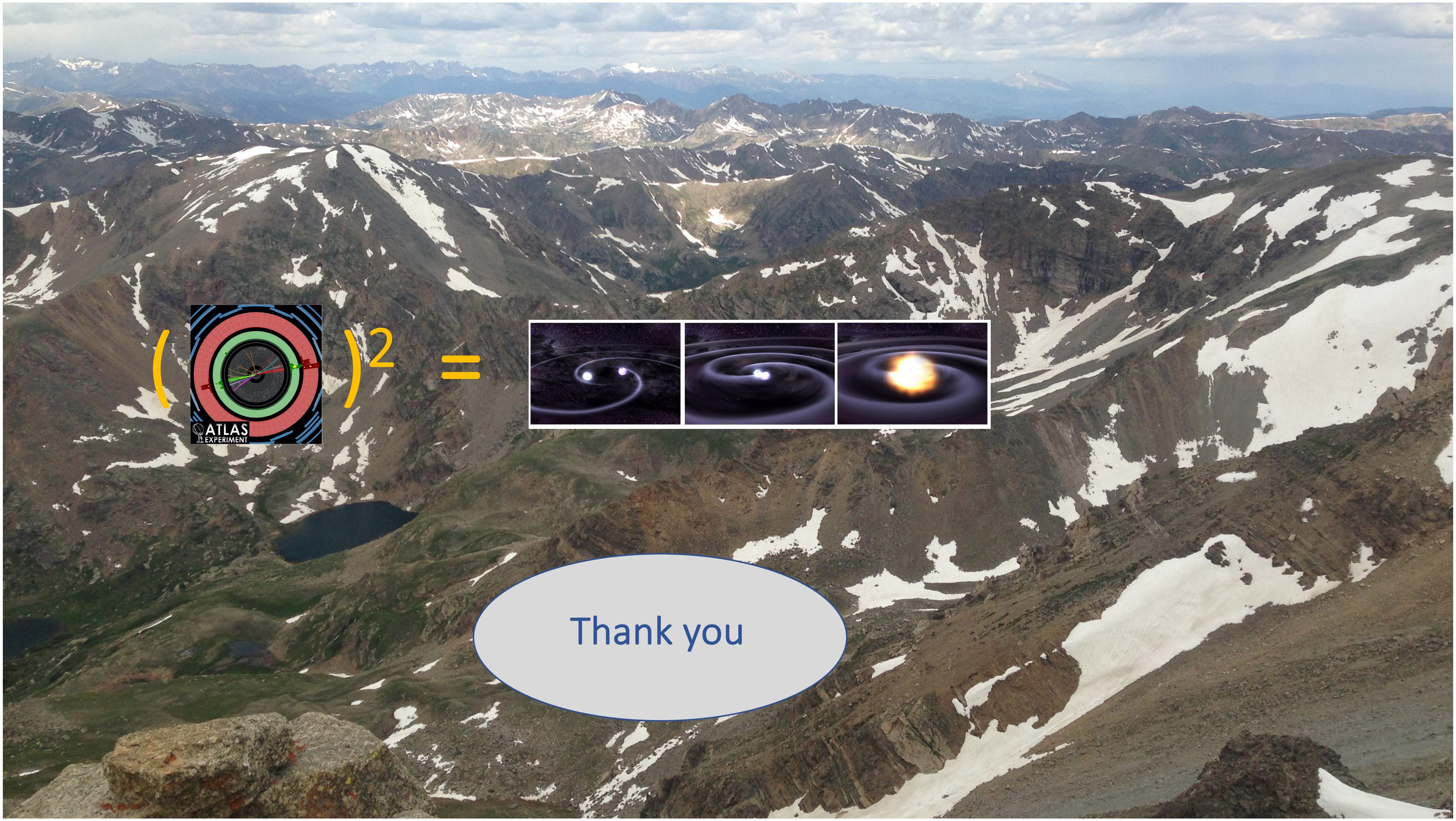

ATLAS
EXPERIMENT
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Thank you

The field theory landscape is incredibly rich

The double-copy is a map among theories that are extremely different:

- **Yang-Mills**: nice renormalizable theory, part of the Standard Model
- **N=4 SYM**: a conformal field theory, widely used in high energy theory
- **gravity**: non-renormalizable, but an amazing EFT!
- **chiral perturbation theory**: low-energy EFT of pions
- **BI** or **sDBI**: low-energy effective actions on D-branes
- **special Galileon**: used in cosmology, but by itself a swampland model

Connected by the "KLT algebra".

The double-copy is part of exploring the space of field theories.