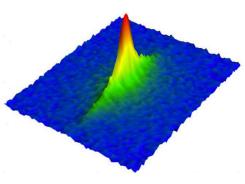
A Framework for Modifying Quantum Mechanics

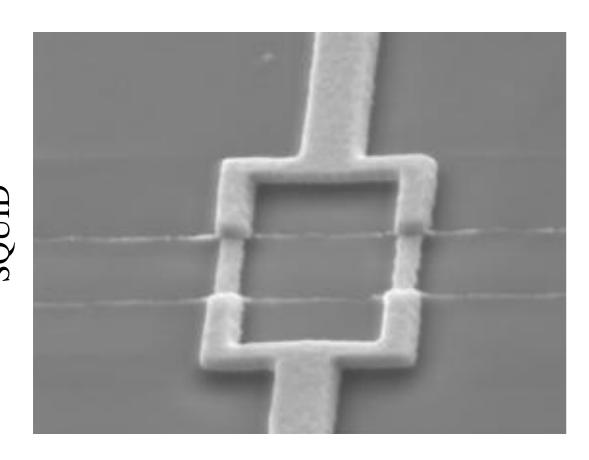
David E. Kaplan

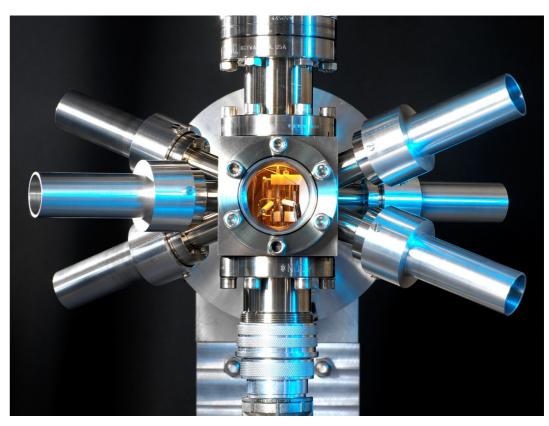
Quantum Mechanics Exploited like never before!

Wave Packet Separation



54 cm

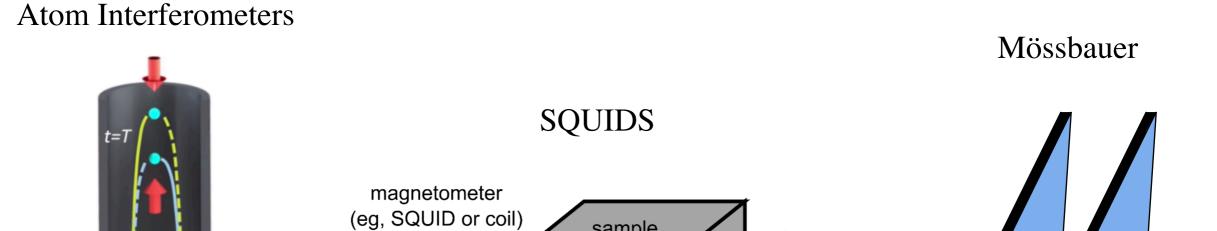


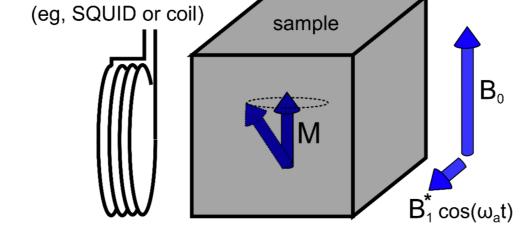


Optical Lattice

Quantum Mechanics

Use to probe fundamental physics!





Gravitational Waves!

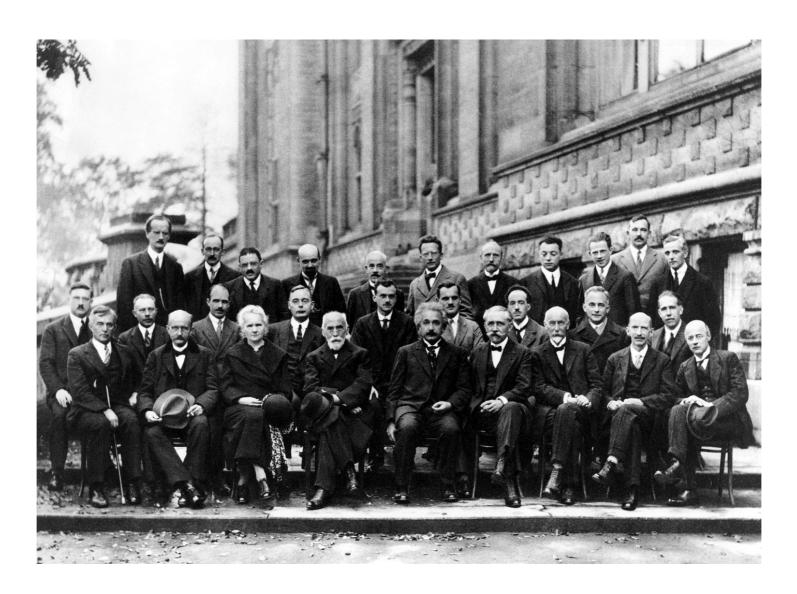
t=2T

Dark Matter!

New Forces!

We can also test quantum mechanics itself!

Quantum Mechanics



1927

Can quantum mechanics be modified or generalized?

Are there sensible theories?

Are there hints of where it might deviate?

What to Modify?

Can we get rid of unsightly 'probability'?

Bell's inequality, etc., rule out local hidden variables.



Ground state of Hydrogen:

If the electron has a definite position, there is an infinite degeneracy of states



What to Modify?

Can we get rid of **linearity**?

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = \hat{H} |\psi\rangle$$

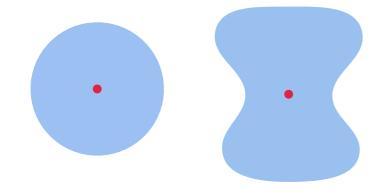
de Broglie (1960) suggested QM is a linear approximation Weinberg (1989) suggested a form

$$\hat{H} \rightarrow \hat{H} \left[|\psi\rangle, \langle\psi| \right]$$

What to Modify?

Can we get rid of **linearity**?

$$\hat{H} \rightarrow \hat{H} \left[|\psi\rangle, \langle\psi| \right]$$



Wave-function self-interaction will move energy levels: check sensitive measurements in QM.



Must check causality and understand measurement.

Can this be embedded in quantum field theory?

Framework

Non-Linear Time Evolution

The Schrödinger Equation: position basis

$$i\hbar \frac{\partial}{\partial t} \psi(x) = H(\mathbf{x}) \, \psi(x)$$

Weinberg's attempt (1989)

$$i\hbar \frac{\partial}{\partial t} \psi(x) = h(\psi^*, \psi) \psi(x)$$

Polchinski showed action at a distance with EPR pairs (1990)

Non-Linear Quantum Mechanics

(warning... c = 1 and $\hbar = 1$)

$$i\frac{\partial}{\partial t}\psi(x) = \hat{H}(\mathbf{x})\,\psi(x) + \epsilon \int d^4x' |\psi(x')|^2 G_R(x'-x)\,\psi(x)$$

Causality guaranteed by the retarded Green's Function (e.g., massless):

$$\Box G_R(y;x) = \delta^4(x-y)$$

$$G_R(x - x') = \frac{\delta \left(t' - (t + |\mathbf{x}' - \mathbf{x}|) \right)}{|\mathbf{x}' - \mathbf{x}|}$$

Non-Linear Quantum Mechanics

One-particle, non-relativistic limit - the non-linear Schrödinger Equation

$$i\frac{\partial}{\partial t}\psi(x) = \hat{H}(\mathbf{x})\,\psi(x) + \epsilon \int d^4x' |\psi(x')|^2 G_R(x'-x)\,\psi(x)$$

$$\underbrace{\delta H(x)}$$

This term is Hermitian, thus the norm conserved

New stationary states can be found perturbatively for a fixed potential

Entangled Systems and Causality

Wave function for two entangled particles:

$$\psi(\mathbf{x}, \mathbf{y}; t) = \sum_{i,j} c_{ij}(t) \psi_i^I(\mathbf{x}) \psi_j^{II}(\mathbf{y})$$

$$\delta \hat{H} \psi = \int d^3x' d^3y' dt' |\psi(\mathbf{x}', \mathbf{y}'; t')|^2 \left(G_R(\mathbf{x}, t; \mathbf{x}', t') + G_R(\mathbf{y}, t; \mathbf{x}', t') + G_R(\mathbf{x}, t; \mathbf{y}', t') + G_R(\mathbf{y}, t; \mathbf{y}', t') \right) \psi(\mathbf{x}, \mathbf{y}, t)$$
causal

$$\delta \hat{H} \subset \int \underline{d^3 x' d^3 y' dt' |\psi(\mathbf{x}', \mathbf{y}'; t')|^2} G_R(\mathbf{y}, t; \mathbf{y}', t')$$

after measurement at \mathbf{x} , this integral unchanged

Quantum Field Theory

$$i \partial_t |\chi\rangle = \hat{H} |\chi\rangle$$

In the Schrödinger picture, the time evolution operator is still:

$$\hat{U} = e^{-i\hat{H}t}$$

with
$$\hat{H} = \int d^3x \ \hat{\mathcal{H}}(\mathbf{x})$$
 made up of field operators

Add state-dependent terms.

Quantum Field Theory

'Non-linear' —> state-dependent

$$i\,\partial_t|\chi\rangle = \left[\int d^3x\,\hat{\mathcal{H}}(\mathbf{x}) + \epsilon\left(\langle\chi|\,\hat{\mathcal{O}}_1(\mathbf{x})\,|\chi\rangle\,\hat{\mathcal{O}}_2(\mathbf{x}) + \hat{\mathcal{O}}_1(\mathbf{x})\,\langle\chi|\,\hat{\mathcal{O}}_2(\mathbf{x})\,|\chi\rangle\right)\right]|\chi\rangle$$

Time evolution includes terms that depends on the state itself

If \mathcal{O}_1 and \mathcal{O}_2 are Hermitian, the norm is constant

$$\partial_t \langle \chi | \chi \rangle = 0$$

Probabilistic interpretation of observables can be maintained

QFT Examples

YUKAWA THEORY

$$\mathcal{H} \supset y \ \phi(\mathbf{x}) \ \overline{\Psi}(\mathbf{x}) \Psi(\mathbf{x})$$

Add non-linearity

$$\langle \phi \rangle \equiv \langle \chi | \phi | \chi \rangle$$

$$\mathcal{H} \supset y (\phi(\mathbf{x}) + \epsilon \langle \phi(\mathbf{x}) \rangle) \bar{\Psi}(\mathbf{x}) \Psi(\mathbf{x})$$

Assuming $\langle \chi | \chi \rangle = 1$

Perturbatively, compute background source in the $\epsilon = 0$ theory

QFT Examples

QED

$$\mathcal{L} \supset eA_{\mu}J^{\mu}$$

Add non-linearity

$$\mathcal{Z} \supset e \left(\frac{A_{\mu} + \epsilon_{\gamma} \langle A_{\mu} \rangle}{1 + \epsilon_{\gamma}} \right) J^{\mu}$$

 A_{μ} and $\langle A_{\mu} \rangle$ have the same gauge transformations — gauge fix and generate the Hamiltonian

GRAVITY

Replace
$$g_{\mu\nu} \to \frac{g_{\mu\nu} + \epsilon_G \langle g_{\mu\nu} \rangle}{1 + \epsilon_G}$$
 in interaction terms. Remains a tensor.

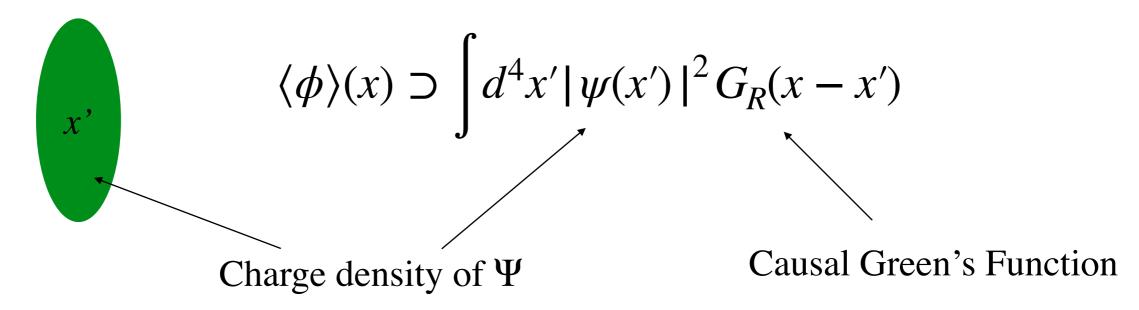
Non-Relativistic Limit — One Particle

$$\mathcal{L} \supset y(\phi + \epsilon \langle \chi | \phi | \chi \rangle) \bar{\Psi} \Psi$$

To get NR theory for fermions Ψ , compute $\langle \phi \rangle$.

Will depend on initial conditions and sources. At zeroth order, Ψ sources ϕ :

 ψ wave function for single fermion Ψ



The Classical Limit

Classical Physics from QM

$$i\hbar\frac{\partial}{\partial t}\langle\hat{\mathcal{O}}\rangle = \left\langle \left[\hat{H},\hat{\mathcal{O}}\right]\right\rangle$$

This leads to
$$\frac{\partial \langle p \rangle}{\partial t} = -\left\langle \frac{\partial V}{\partial x} \right\rangle$$
 and $\frac{\partial \langle x \rangle}{\partial t} = \frac{\langle p \rangle}{m}$

Or, F = ma on average

Coherent states (or classical-like states) are

ones in which, e.g.,
$$\left\langle \frac{\partial V(x)}{\partial x} \right\rangle \simeq \frac{\partial V(\langle x \rangle)}{\partial \langle x \rangle}$$

Classical Physics from NLQM

Say
$$\mathcal{H} \supset \hat{A}_{\mu}\hat{J}^{\mu} + \epsilon_{\gamma} \left(\langle \hat{A}_{\mu} \rangle \hat{J}^{\mu} + \hat{A}_{\mu} \langle \hat{J}^{\mu} \rangle \right)$$

$$i\hbar\frac{\partial}{\partial t}\langle\hat{\mathcal{O}}\rangle = \int\langle\left[\hat{\mathcal{H}},\hat{\mathcal{O}}\right]\rangle\supset\int\langle\left[\hat{A}\cdot\hat{J},\hat{\mathcal{O}}\right]\rangle + \epsilon_{\gamma}\langle\left[\langle\hat{A}\rangle\cdot\hat{J},\hat{\mathcal{O}}\right]\rangle + \epsilon_{\gamma}\langle\left[\hat{A}\cdot\langle\hat{J}\rangle,\hat{\mathcal{O}}\right]\rangle$$

$$= \int \left(\langle \hat{A} \cdot \left[\hat{J}, \hat{\mathcal{O}} \right] \rangle + \epsilon_{\gamma} \langle \hat{A} \rangle \cdot \langle \left[\hat{J}, \hat{\mathcal{O}} \right] \rangle \right) + \left(\langle \left[\hat{A}, \hat{\mathcal{O}} \right] \cdot \hat{J} \rangle + \epsilon_{\gamma} \langle \left[\hat{A}, \hat{\mathcal{O}} \right] \rangle \cdot \langle \hat{J} \rangle \right)$$

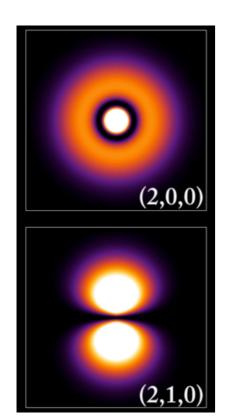
The point is, $\langle \hat{A}_{\mu} \hat{J}^{\mu} \rangle \simeq \langle \hat{A}_{\mu} \rangle \langle \hat{J}^{\mu} \rangle$ for classical states

In fact, the non-linear terms make physics more classical!

Constraints from Quantum Systems

Atomic levels — lamb shift, (g-2) of the electron, ...

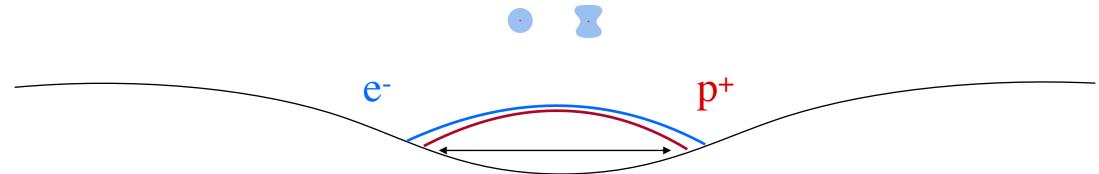
Atomic Levels



What does this do to the Lamb Shift?

Say charged particles see their own w.f.:

$$i\frac{\partial}{\partial t}\psi(x) = H(\mathbf{x})\psi(x) + \epsilon_{\gamma}\alpha \int d^4x' |\psi(x')|^2 G_R(x'-x)\psi(x)$$



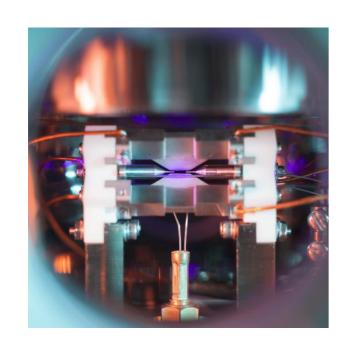
Electron spread over the trap (micron) dilutes the electric field and thus the level splitting Proton's wave function also produces a field that nearly cancels the electron wave function.

Key — center of mass coordinate cannot be separated from relative coordinate due to locality.

$$\epsilon_{\gamma} < 10^{-2}$$

Constraints

Leading Constraint

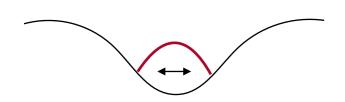


Ion Traps



For $\varepsilon_{\gamma} > 0$ (repulsive interaction)

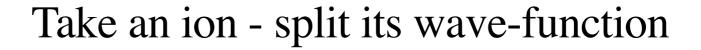
Too large a repulsion, can't trap ion $\varepsilon_{\gamma} < 10^{-5}$



No direct limit on $\varepsilon_{\gamma} < 0$ (attractive interaction) Perhaps from mapping of ion in trap?

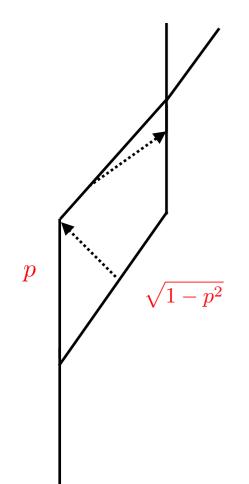
Experimental Tests

Interferometry - interaction between paths



Coulomb Field of one path interacts with the other path

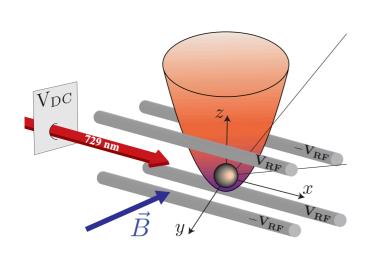
Gives rise to phase shift that depends on the intensity p^2 of the split



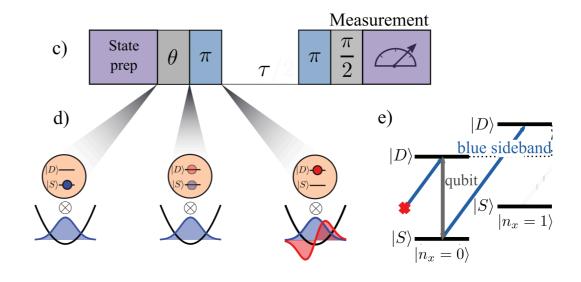
Use intensity dependence to combat systematics

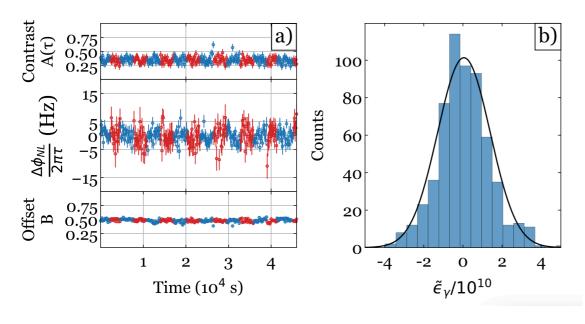
Test ... Using a Vibrational Mode of a Trapped Ion

J. Broz, et al. (2022)



State: $|\psi\rangle = a|0\rangle + b|1\rangle$



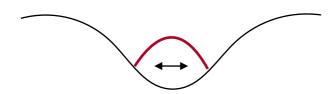


Non-linear phase:
$$\phi_{NL} = \epsilon_{\gamma} \alpha \frac{10a^2 + b^2}{30\sqrt{2\pi}\hbar x_0} \tau$$

$$\epsilon_{\gamma} = 5 \pm 5.4 \times 10^{-12}$$

Experimental Tests

Atomic Aging



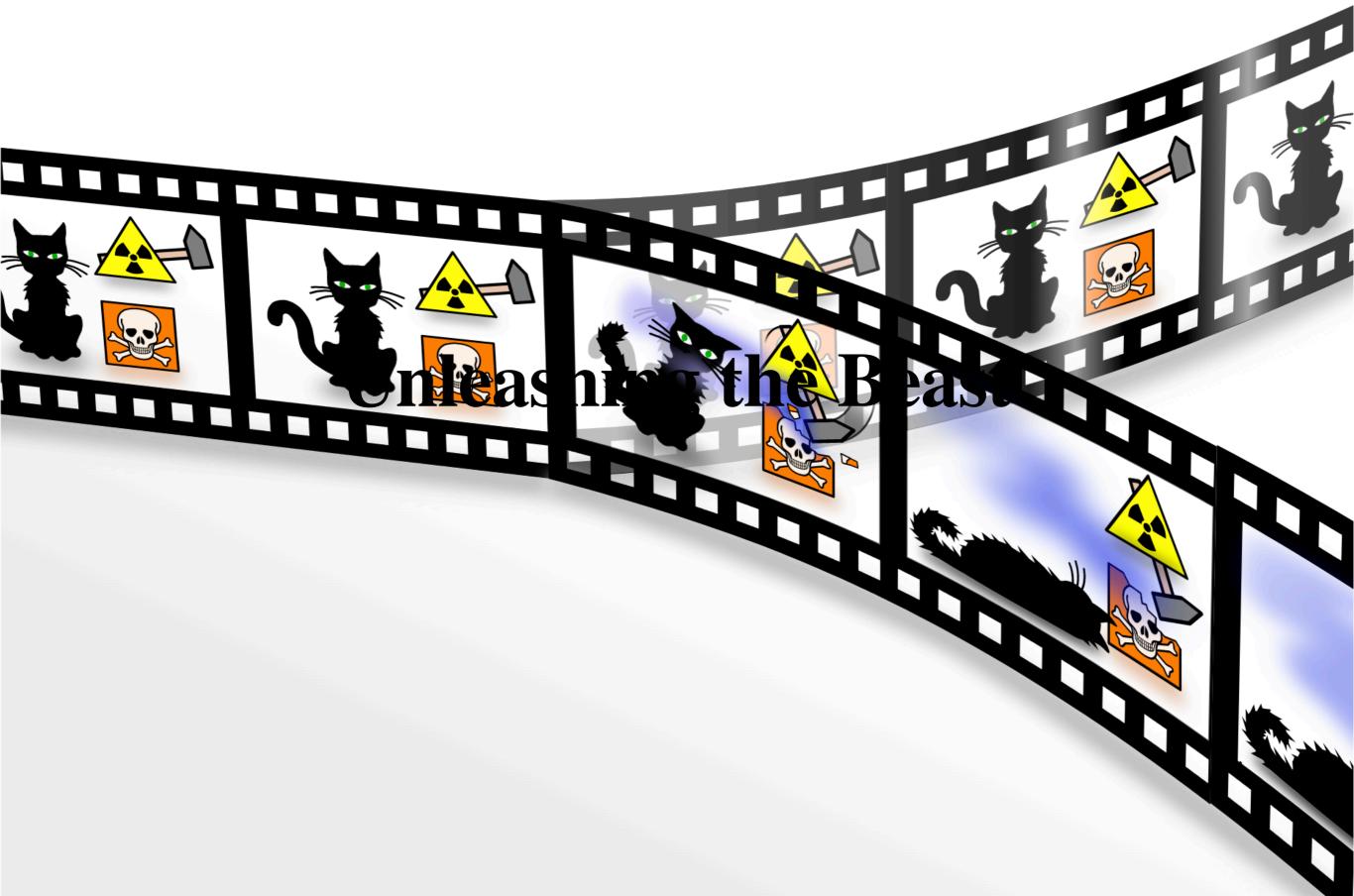
Ion Trap with decaying nucleus — wave function evolves over time:

$$|\psi\rangle = e^{-\Gamma t/2} |\text{Trapped}\rangle + \sqrt{1 - e^{-\Gamma t}} |\text{Decayed}\rangle$$



Background wave function produces E-field in trap and second order Stark shift in the atomic states. $\delta \phi \sim E_{\rm NI}^2 \Delta \alpha_P T$

(not ideal as background suppression suppresses this effect)



Measurement

Stern-Gerlach Experiment



B-Field

$$\hat{H} = \hat{\mu} \cdot \mathbf{B} + \frac{\hat{p}^2}{2m} + \cdots$$

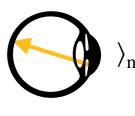
$$|\psi(t_0)\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle) \otimes |\text{everything else}\rangle$$

$$|\psi(t_1)\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle| + p\rangle + |\downarrow\rangle| - p\rangle) \otimes |\text{everything else}\rangle$$

$$|\psi(t_2)\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle| + p\rangle |\text{upper pixel}\rangle_{\text{screen}} + |\downarrow\rangle| - p\rangle |\text{lower pixel}\rangle_{\text{screen}}) \otimes |\text{everything else}\rangle$$

$$|\psi(t_3)\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle| + p\rangle |\text{upper pixel}\rangle_{\text{screen}}|$$
 $\rangle_{\text{me}} + |\downarrow\rangle| - p\rangle |\text{lower pixel}\rangle_{\text{screen}}|$





Measurement in Quantum Mechanics

Time evolution with interaction between the system and measuring device

$$|\chi\rangle \otimes |A_0\rangle \rightarrow \sum_i c_i |i\rangle \otimes |A_i\rangle$$

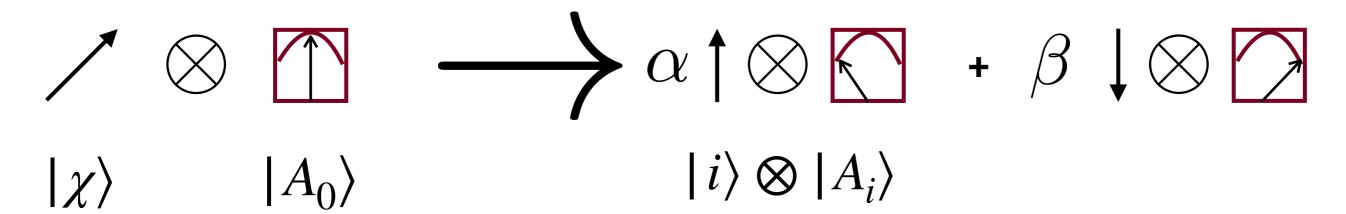
Prediction of Quantum Mechanics ("Many Worlds")

Pick:
$$\langle A_j | A_i \rangle = \delta_{ij} \Longrightarrow \rho_{|\Psi\rangle} = \sum_i c_i c_i^* |i\rangle \langle i|$$

"Interpret" as direct sum of "worlds"

Measurement in Quantum Mechanics

Time evolution with interaction between the system and measuring device



In linear QM, can pick orthogonal basis vectors just by knowing the interaction Hamiltonian

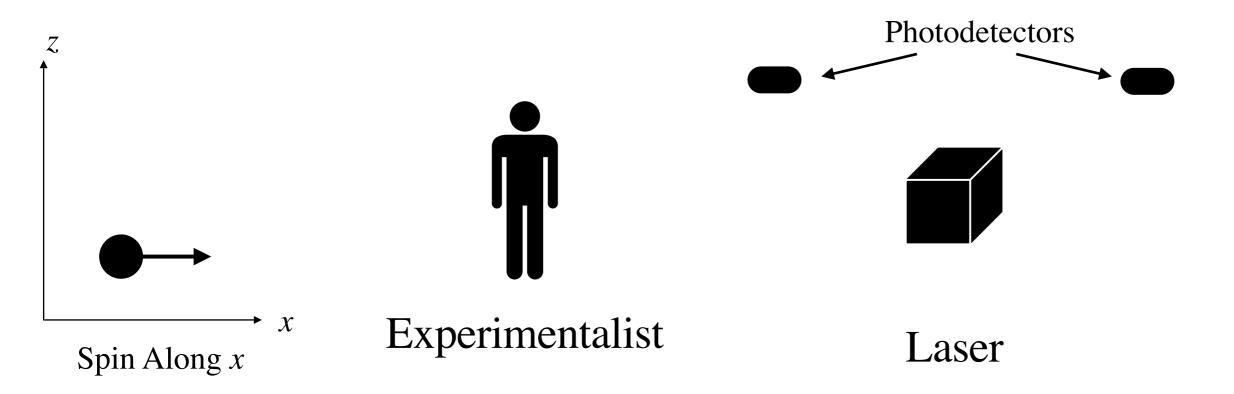
$$\langle A_j | A_i \rangle = \delta_{ij}$$

In non-linear QM, stationary states are generally **not** orthogonal — the effective Hamiltonian depends on the initial state of the system

No Guarantee:
$$\langle A_j | A_i \rangle = 0$$

$$|\Psi\rangle\otimes|A_0\rangle \to \sum_i c_i|i\rangle\otimes|A_i\rangle + \epsilon\sum_{i,j} d_{i,j}|i\rangle\otimes|A_j\rangle$$
 Measurement noise

Linear Quantum Mechanics



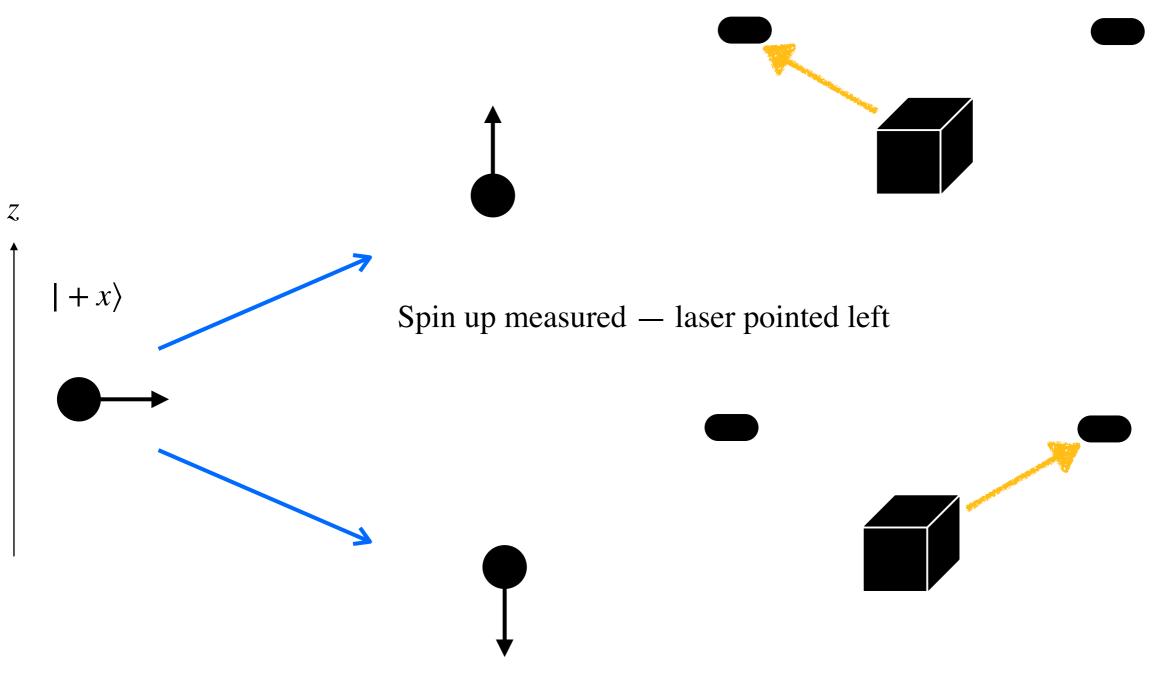
Initial State : $|\chi(0)\rangle$

Represents Full Quantum State (spin, experimentalist...)

Goal: Create Macroscopic Superposition

Method: Measure spin along *z*. Depending upon outcome, send laser along different directions

Linear Quantum Mechanics



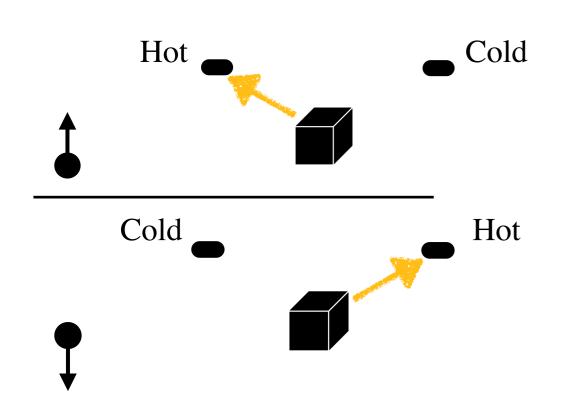
Spin down measured — laser pointed right

$$|\chi(t)\rangle = \frac{1}{\sqrt{2}} \left(|+z\rangle |L\rangle |Env_L\rangle + |-z\rangle |R\rangle |Env_R\rangle \right)$$

Linear Quantum Mechanics

Which photodetectors light up?

$$\mathcal{H} \supset eA_{\mu}J^{\mu}$$



Transition Matrix Elements

$$\langle +z | \langle L | \langle Env_L | A_{\mu}(x_L) J^{\mu}(x_L) | +z \rangle | L \rangle | Env_L \rangle \neq 0$$

$$\langle +z | \langle L | \langle Env_L | A_{\mu}(x_R) J^{\mu}(x_R) | +z \rangle | L \rangle | Env_L \rangle = 0$$

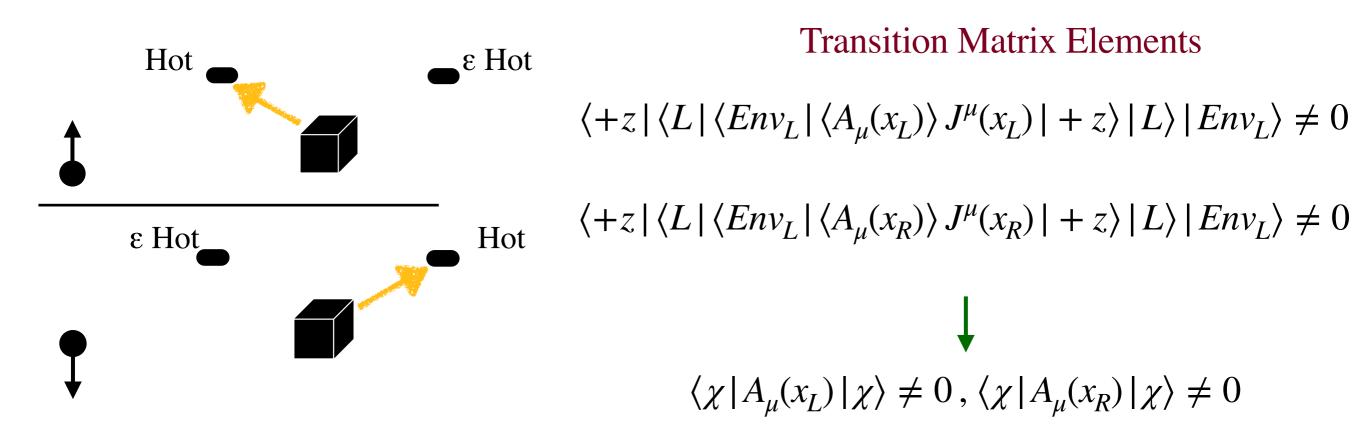
$$\langle L | A_{\mu}(x_R) | L \rangle = 0$$

$$|\chi(t)\rangle = \frac{1}{\sqrt{2}} \left(|+z\rangle |L\rangle |Env_L\rangle + |-z\rangle |R\rangle |Env_R\rangle \right)$$

Non-Linear Quantum Mechanics

Which photodetectors light up?

$$\mathcal{H} \supset eA_{\mu}J^{\mu} + e\varepsilon \langle A_{\mu} \rangle J^{\mu}$$

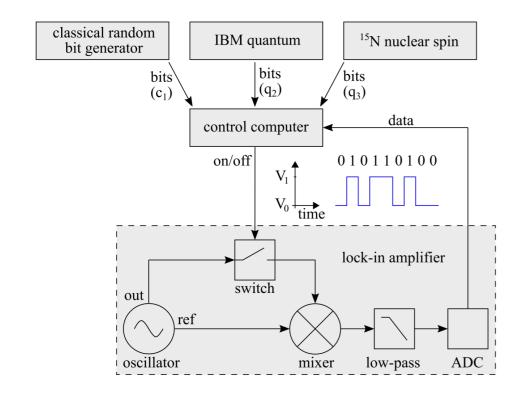


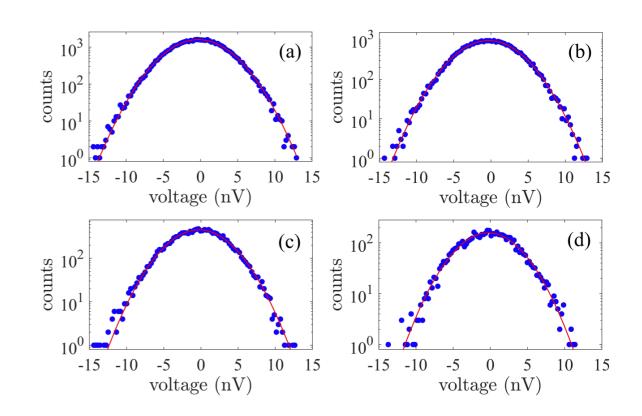
Communication between "worlds"

Non-linearity visible despite Environmental De-coherence! Polchinski: "Everett Phone"

Experimental limit on non-linear QM using a voltmeter and quantum bits

M. Polkovnikov, et al (2022)



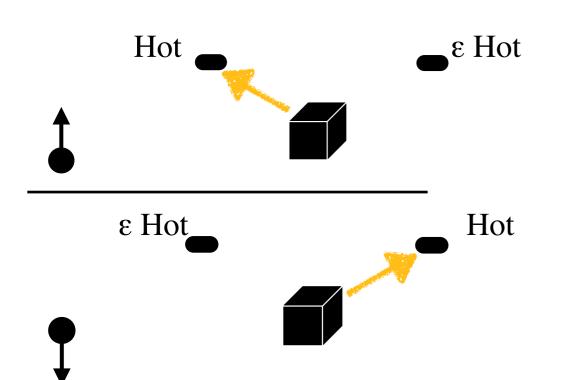


$$|\epsilon_{\gamma}| \le 4.7 \times 10^{-11}$$

Quantum Pollution

Delicate Non-Linearity

O performs the laser experiment on October 24th - discovers non-linear quantum mechanics!



$$|\chi\rangle = \frac{1}{\sqrt{2}} \left(|L\rangle |O_L\rangle + |R\rangle |O_R\rangle \right)$$

Now O wants to repeat experiment

Suppose $|O_U\rangle$ decides to run experiment at 9am on Oct 26 But $|O_D\rangle$ runs experiment on 9am on Nov 3rd

State just after 9am on Oct 26

$$|\chi\rangle = \frac{1}{\sqrt{2}} \left(|L\rangle |O_L\rangle \frac{|L'\rangle |O_L'\rangle + |R'\rangle |O_D'\rangle}{\sqrt{2}} + |R\rangle |O_R\rangle \right)$$

Delicate Non-Linearity

State after 9am on Oct 26

Compare with State on Oct 24

$$|\chi\rangle = \frac{1}{\sqrt{2}} \left(|L\rangle |O_L\rangle \frac{|L'\rangle |O_L'\rangle + |R'\rangle |O_D'\rangle}{\sqrt{2}} + |R\rangle |O_R\rangle \right) \qquad |\chi\rangle = \frac{1}{\sqrt{2}} \left(|L\rangle |O_L\rangle + |R\rangle |O_R\rangle \right)$$

$$\langle L | \langle O_L | \langle L' | \langle O_L' | \langle A_\mu(x_R) \rangle J^\mu(x_R) | \chi(t = \text{Oct 26}) \rangle = \frac{1}{2} \langle L | \langle O_L | \langle A_\mu(x_R) \rangle J^\mu(x_R) | \chi(t = \text{Oct 24}) \rangle$$

Effect is 1/2 of prior effect!

But, full effect if O_U and O_D perform experiment at same time!

Quantum Pollution: Without adequate care, superpositions may diverge wildly, preventing exploitability. Not automatic - but need careful protocols!

But hasn't there already been dilution?

What part of the wave function...

$$|\chi\rangle = \alpha |\text{Us}\rangle + \beta |\text{Them}\rangle$$

$$\mathcal{H} \supset eA_{\mu}J^{\mu} + e\varepsilon \langle A_{\mu} \rangle J^{\mu}$$

$$|\chi\rangle = \alpha |\operatorname{Us}\rangle + \beta |\operatorname{Them}\rangle \to \langle \chi |A_{\mu}|\chi\rangle = |\alpha|^{2} \langle U |A_{\mu}|U\rangle + |\beta|^{2} \langle T |A_{\mu}|T\rangle$$
$$\langle \chi |A_{\mu}|\chi\rangle \to |\alpha|^{2} \langle U |A_{\mu}|U\rangle$$

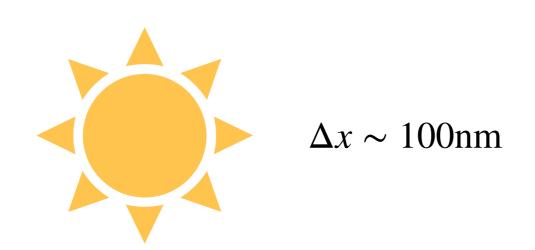
For $\alpha \ll \beta$, the wave function is dominated by something we are not a part of. Can't turn on coherent fields over there.

Local exploitability completely determined by unchangeable initial conditions dramatic difference from linear QM

Classical World?

$$|U(t)\rangle = |\diamondsuit\rangle + \delta| \blacksquare \rangle$$
 Or $|U(t)\rangle = \delta|\diamondsuit\rangle + |\blacksquare\rangle$

Are there natural quantum amplifiers, for e.g. in chaotic systems?



Changing classical evolution of a system requires coherent motion of N atoms

Probability that N atoms coherently move in some way: p^N

With p \sim O(1) scattering probability

What about the weather? What about my brain??



100's + ions to get one neuron to fire

$$|U(t)\rangle = | \langle \rangle \rangle + \delta | \square \rangle$$
 Reasonable

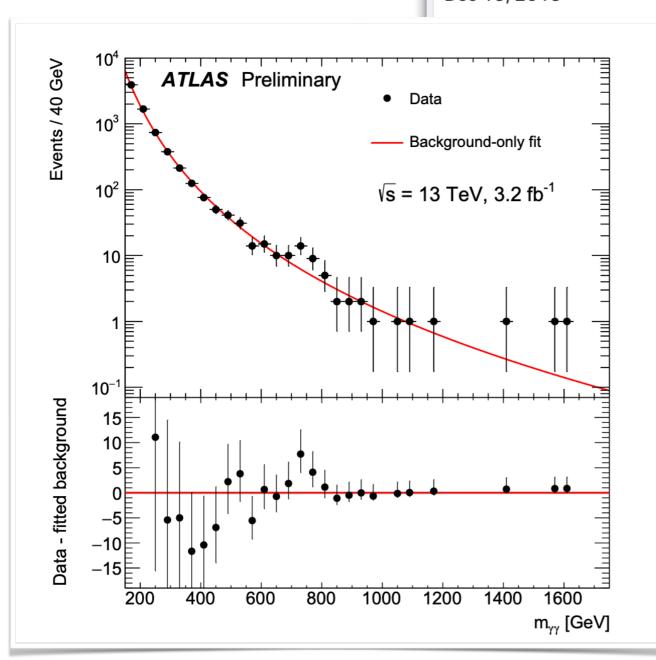
Quantum Amplifiers are Hard!

Natural Quantum Dilution

The '750 GeV' resonance!

Search for resonances decaying to photon pairs in 3.2 fb $^{-1}$ of pp collisions at \sqrt{s} = 13 TeV with the ATLAS detector

Dec 15, 2015



NF-2015-081
NTLAS
Server

claim

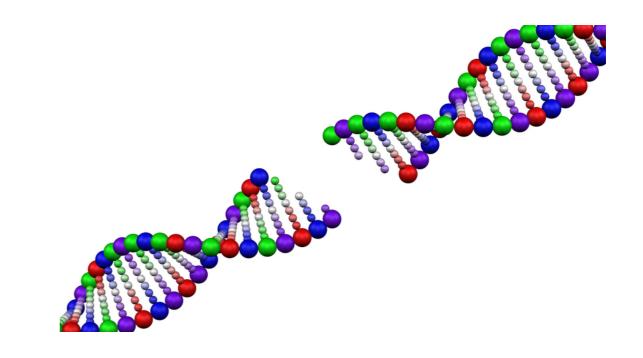
☐ reference search → 562 citations

Have we been diluting our wave function on Earth for the past 100 years?

Evolutionary Dilution?

Is $N \sim O(\text{few})$ for evolution? Maybe for RNA/DNA? RNA formation?

Evolution in an amplifier!



$$|U(t=0)\rangle = |U(t=0)\rangle = |U(t=0)\rangle = |U(t)\rangle = |U(t)\rangle = |U(t)\rangle = |U(t)\rangle + |V(t)\rangle = |U(t)\rangle + |V(t)\rangle = |U(t)\rangle + |V(t)\rangle = |U(t)\rangle + |V(t)\rangle + |V(t)\rangle = |U(t)\rangle + |V(t)\rangle + |V(t$$

$$|U(t)\rangle = |U(t)\rangle = |U(t)\rangle + |\Psi\rangle + \dots$$

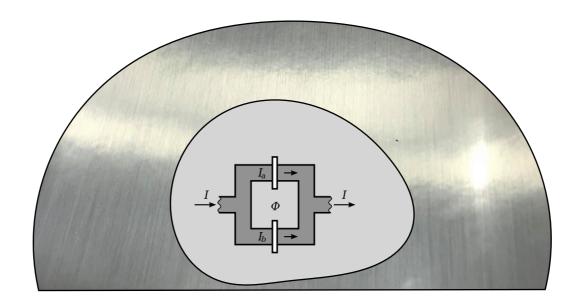
Tests for a Quantum-Diluted Earth

Look for coherent fields turned on in all parts of the wavefunction:

The magnetic field of the Earth!

$$eJ^{\mu}(A_{\mu} + \epsilon_{\gamma}\langle A_{\mu}\rangle)$$

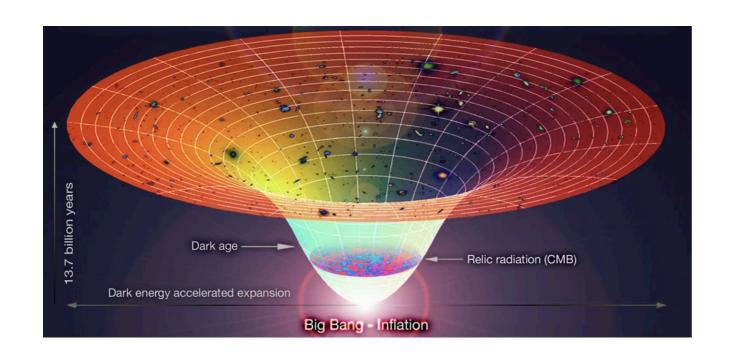
Build a magnetic field structure unique to our part of the wave function and measure the field inside.



$$|U(t)\rangle = |0\rangle (\alpha | \text{shield} \rangle + \beta | \text{No shield} \rangle$$

Cosmological Quantum Amplifier: Inflation

Standard cosmic inflation: rapidly places quantum state in a homogenous and isotropic state (Bunch-Davies Vacuum)



How could homogeneous state become inhomogeneous?

Answer: Massive Superposition of Statistically Similar Universes!

$$|\chi\rangle = \sum_i c_i |U_i\rangle, \ c_i \sim e^{-N}$$

Most of the Universe: The space-time point where the Earth is located is in intergalactic space!

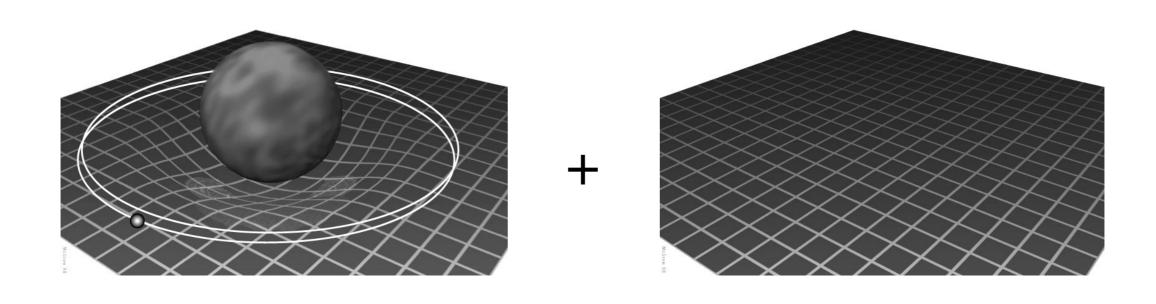
Tests for a Quantum-Diluted Universe(!)

Look for coherent fields turned on in all parts of the wavefunction:

The magnetic field of the Earth!

$$T^{\mu\nu}(g_{\mu\nu} + \epsilon_G \langle g_{\mu\nu} \rangle) + \cdots$$

Objects in our part of the wave function will produce different gravitational fields than the average



Tests for a Quantum-Diluted Universe

$$g_{\mu\nu} \rightarrow \frac{g_{\mu\nu} + \epsilon_G \langle g_{\mu\nu} \rangle}{1 + \epsilon}$$

$$g_s = -\left(1 - \frac{r_s}{r}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} + r^2 d\Omega^2$$
 $\langle g \rangle = -dt^2 + dr^2 + r^2 d\Omega^2$

Renormalize and Expand

$$g_{\text{eff}} \simeq \left[-\left(1 - \frac{R_s}{r}\right) dt^2 + \left(1 + \frac{R_s}{r} + \left(\frac{R_s}{r}\right)^2 \left(1 + \epsilon_G\right)\right) dr^2 \right] + r^2 d\Omega^2$$

Looks like a long-distance modification of gravity!

Corrects second-order GR term —> Strong field tests of GR

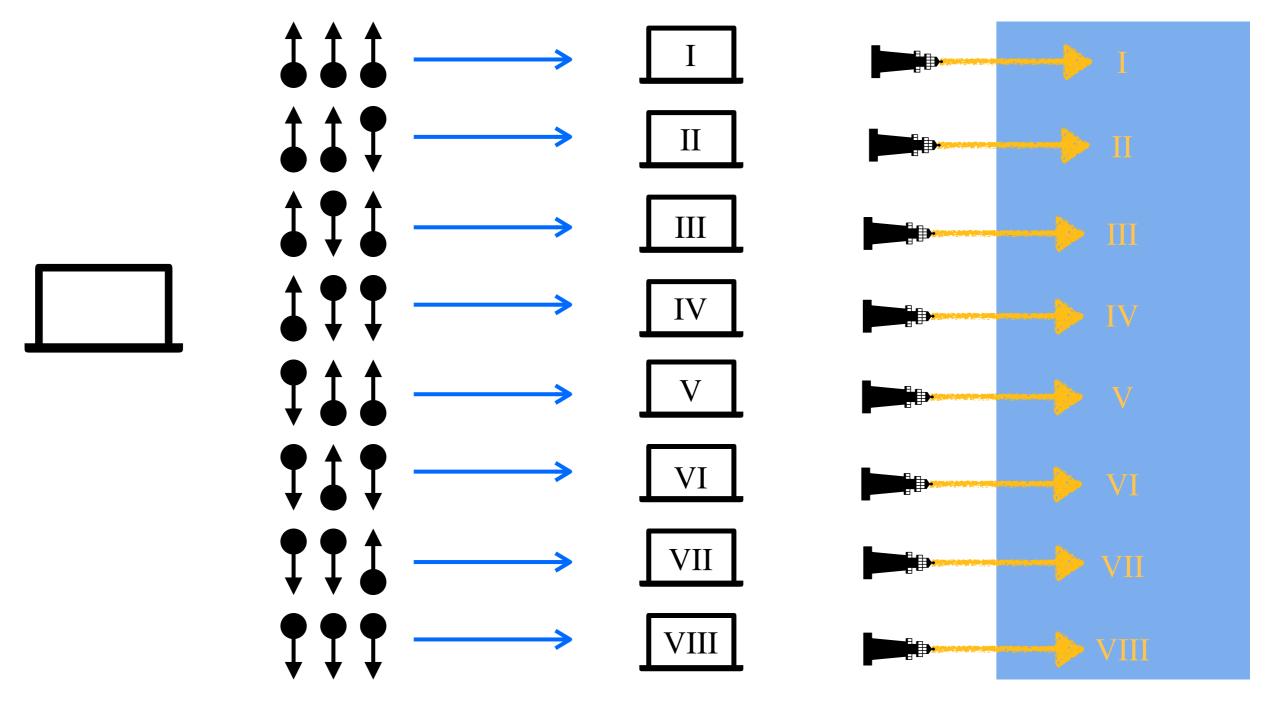
Will potentially make a black hole horizon more singular

Implications

If we have a Classical Universe

Macroscopic superpositions can be produced at will.

Parallelize any computation:



Quantum Computing!

Conclusions

Conclusions

There is a consistent way to explore non-linear deviations from QM

Locality makes many past tests insensitive — new probes required

NL effects can be experimentally tested by amplifying quantum measurements

Quantum amplification in the history of the universe suppresses access to local non-linearities —> Linear QM is an attractor solution.

If locally diluted, non-zero fields across the wave function could be detected (Earth's magnetic field, cosmological metric)

If NLQM is locally accessible, it will radically change what we can do technologically

Thank you!