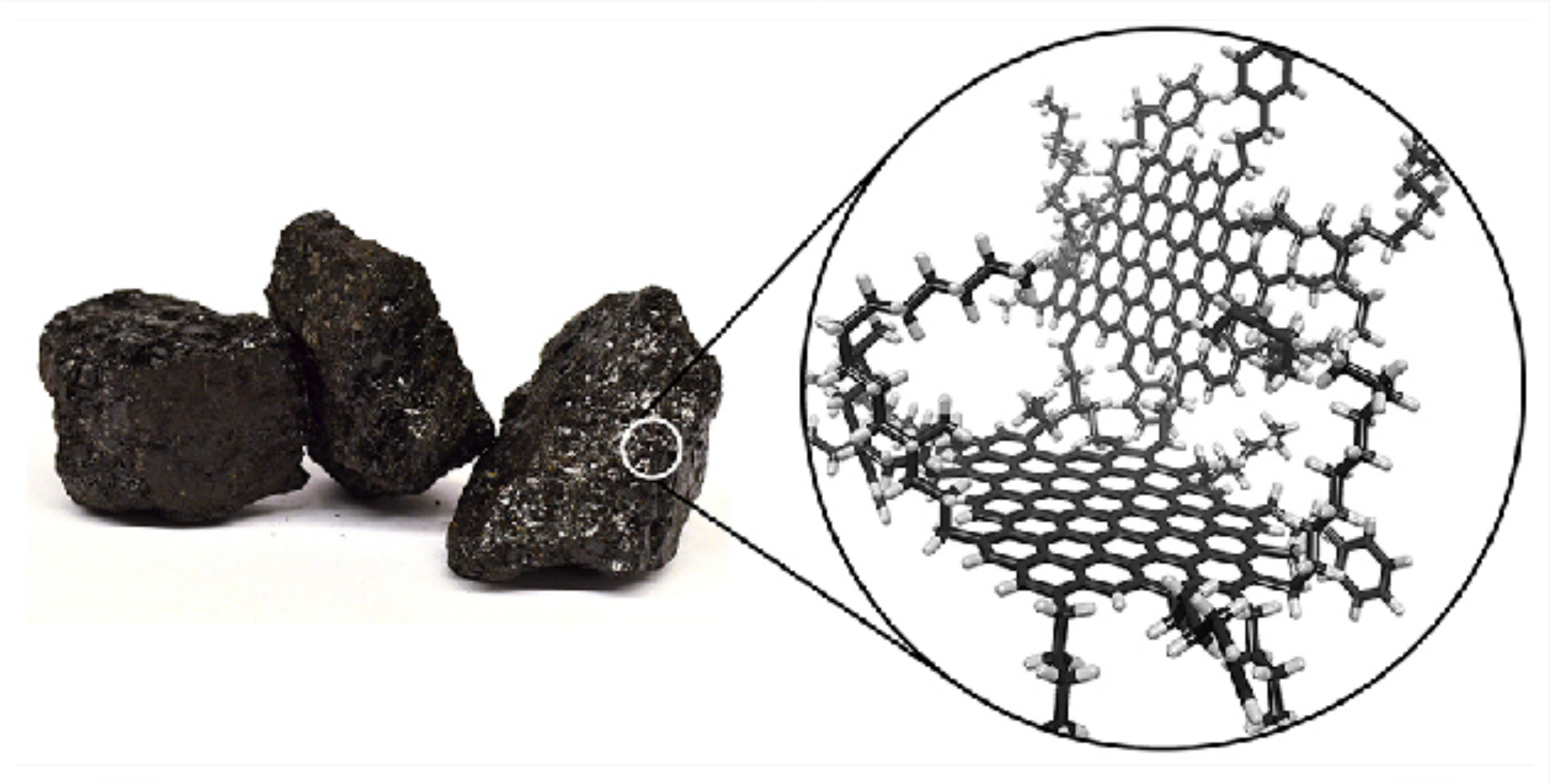
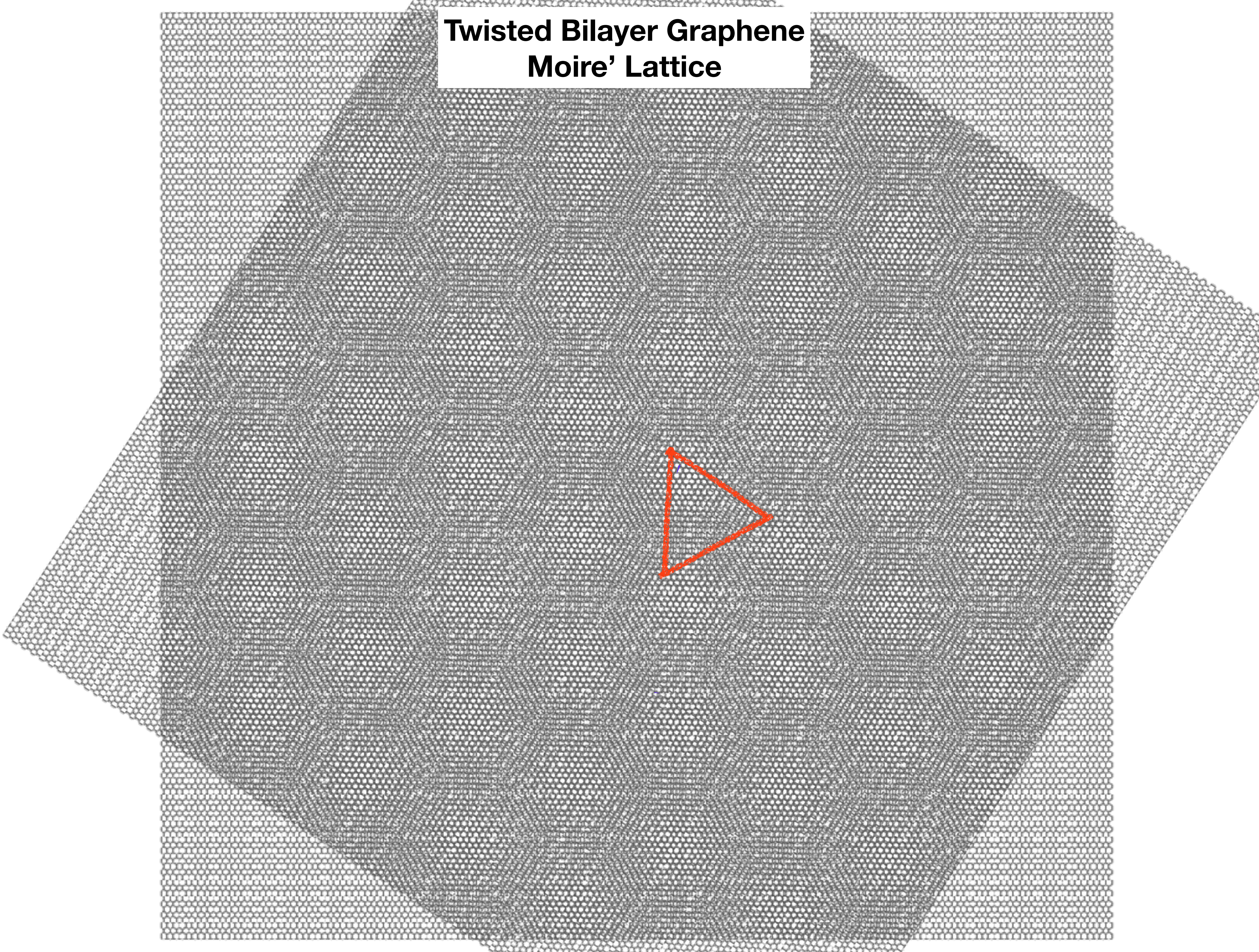


More and Different: Novel Quantum Phases in Moiré Lattices

Ashvin Vishwanath

@Harvard University

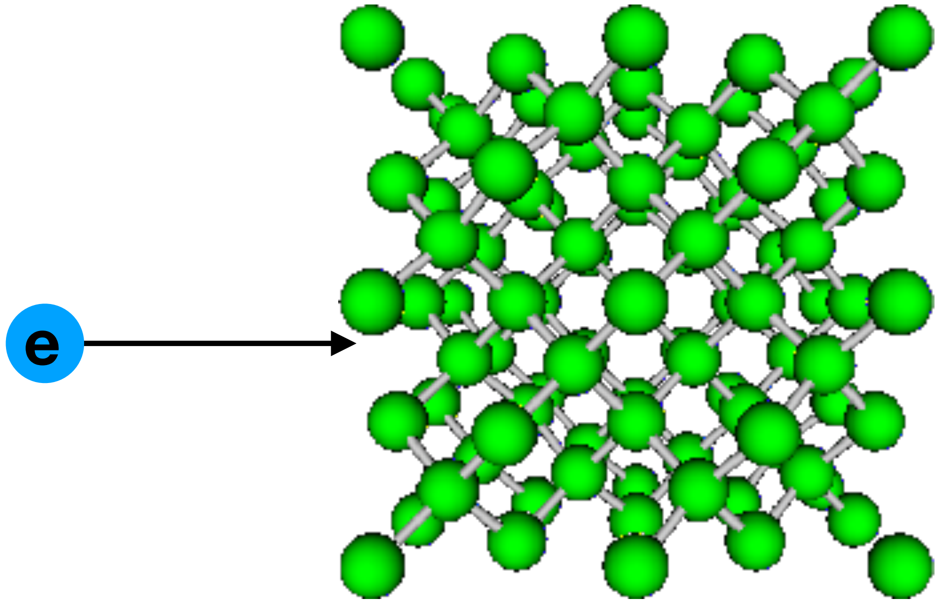
Twisted Bilayer Graphene Moire' Lattice

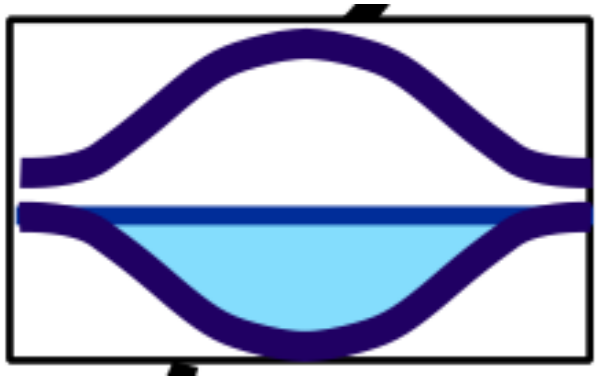


Electronic States in a Crystal

Metals

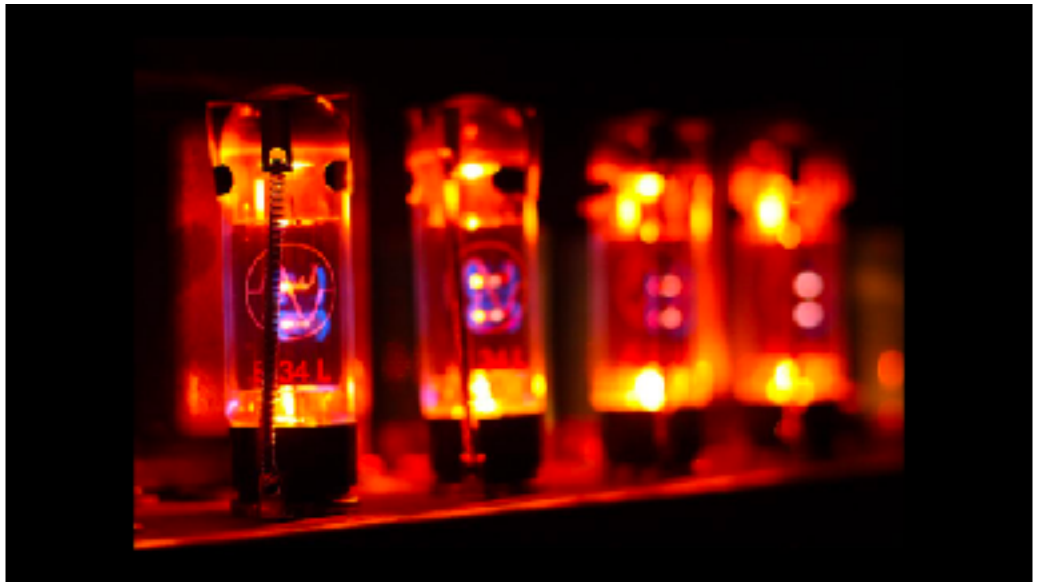
Insulators





Crystal => Artificial Vacuum for the electrons

Crystals - Artificial Vacuum for Electrons

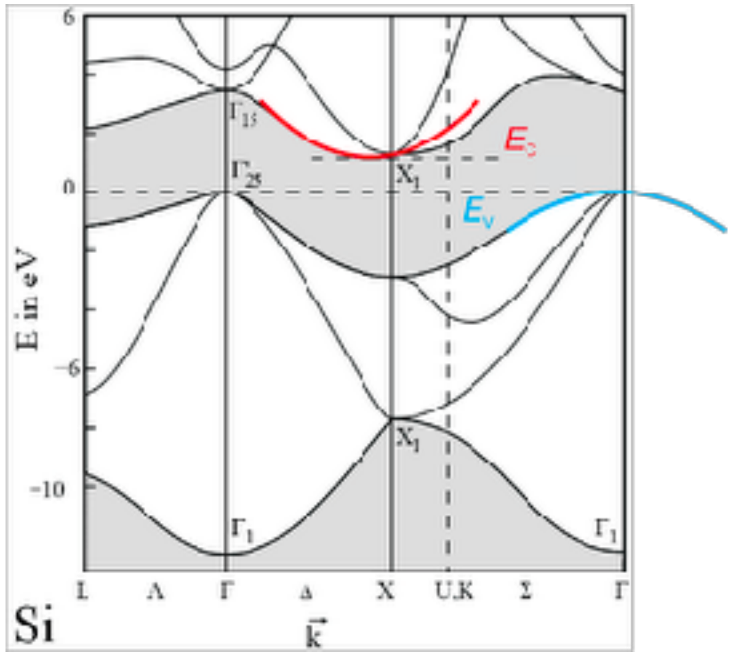


Vacuum Tubes <1960s

Transistor

“A sudden gasp filled the room when he flicked on an oscillator circuit,
and it emitted a shrill tone instantaneously, with no warmup delay whatsoever.”

Demonstration of the Transistor 1948
From - Crystal Fire

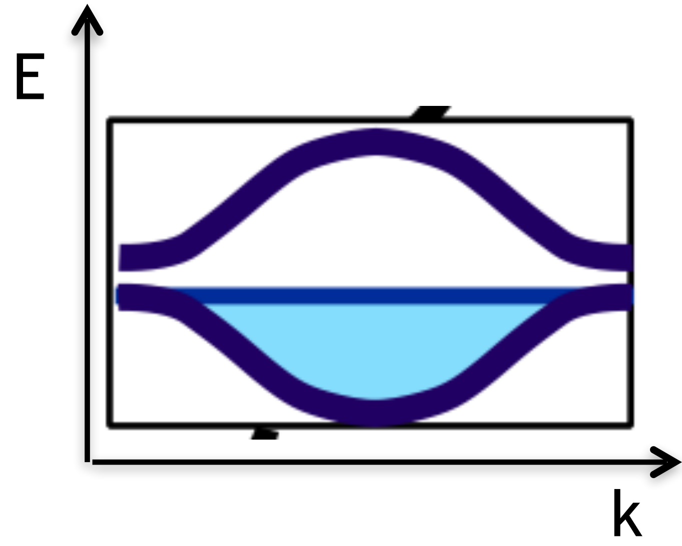


Modify properties of the electron
Effective mass, electrons+holes etc.

Qualitatively new effects?

- Semi-classical theory of electrons in a crystal

$$\dot{x} = \nabla_k \mathcal{E}(k) \quad + \dot{k} \times \tilde{B}$$
$$\dot{k} = -\nabla_r \mathcal{V}(r) + e\dot{x} \times B$$

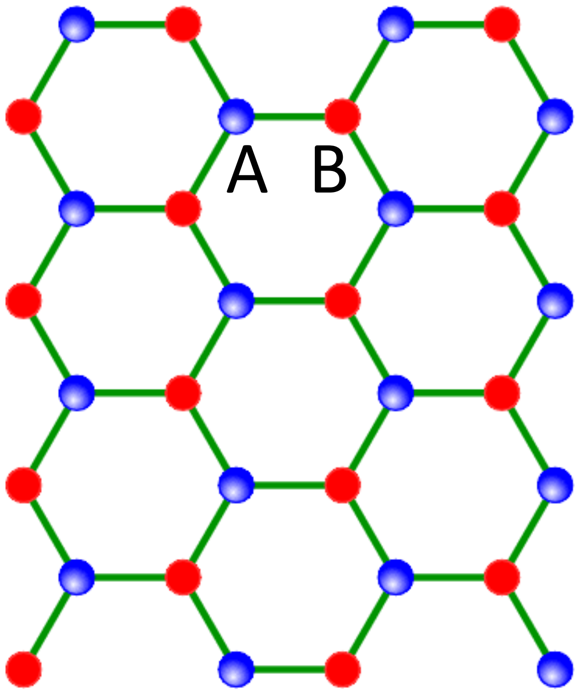


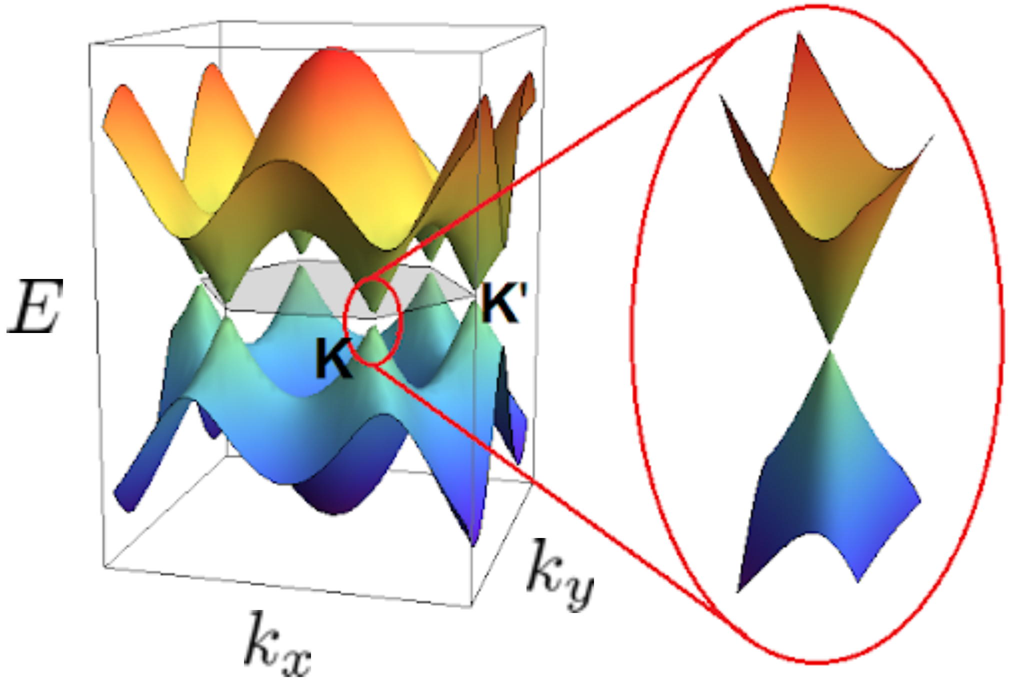
- Symmetry restored in a crystal – Berry Flux $\tilde{B}$ leads to an anomalous velocity.

- Berry Flux is related to the Berry’s phase acquired by states in the band. “Quantum Geometry” of bands.

Qualitatively new effects in band structures

Graphene band structure





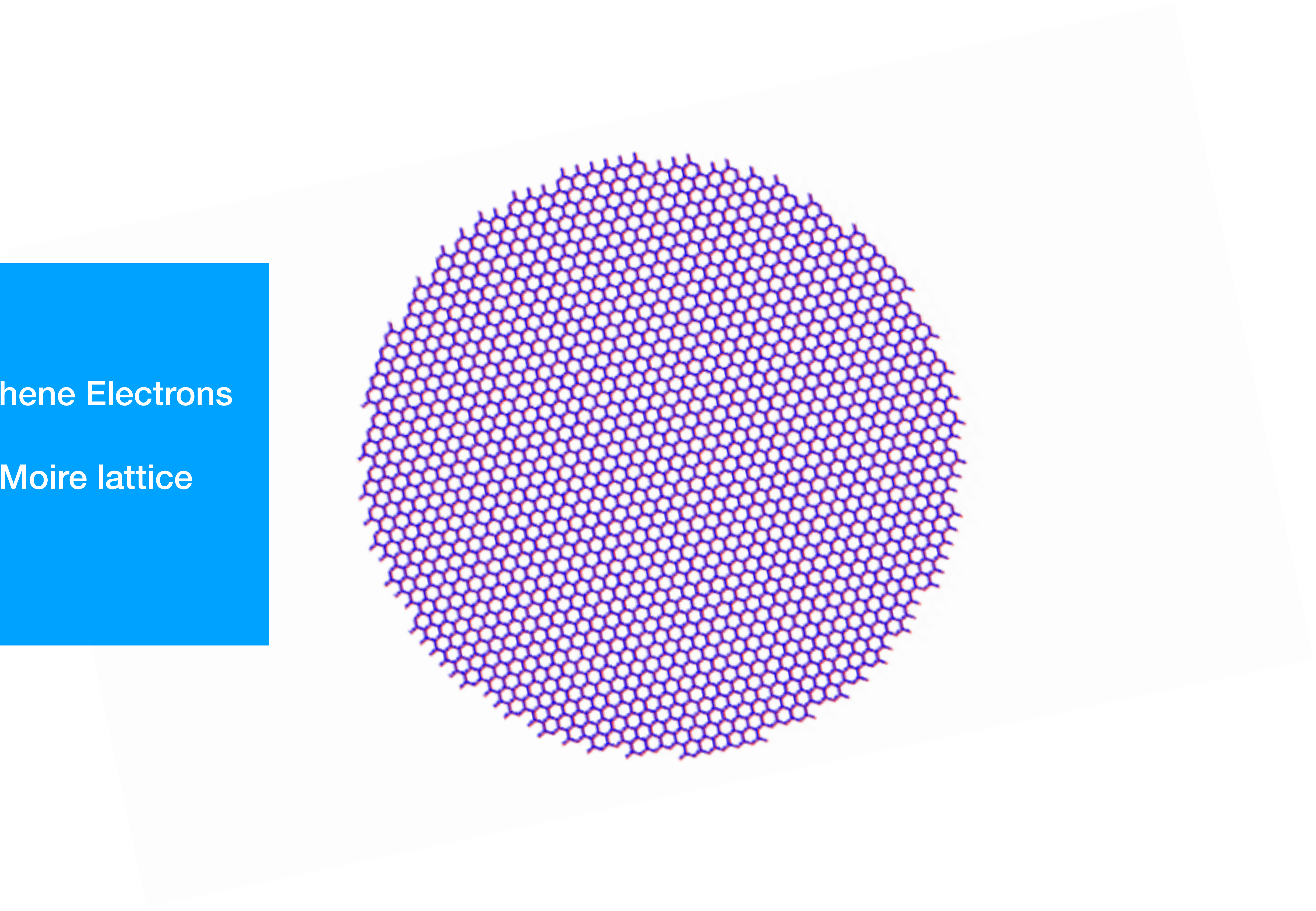
Electrons governed by 2+1D *massless* Dirac equation!

$$\Psi = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} \qquad H = v_F \hat{\alpha} \cdot p$$

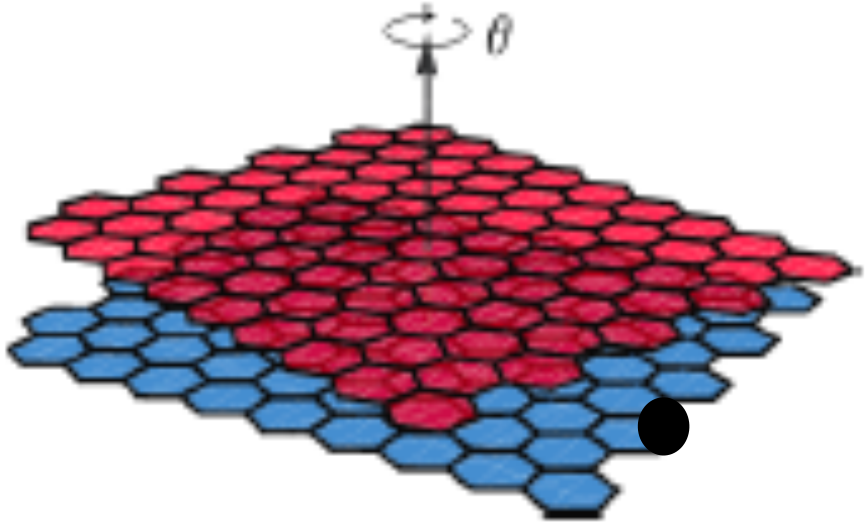
Graphene’s Electrons in Moiré lattice

Graphene Electrons

\+ Moire lattice

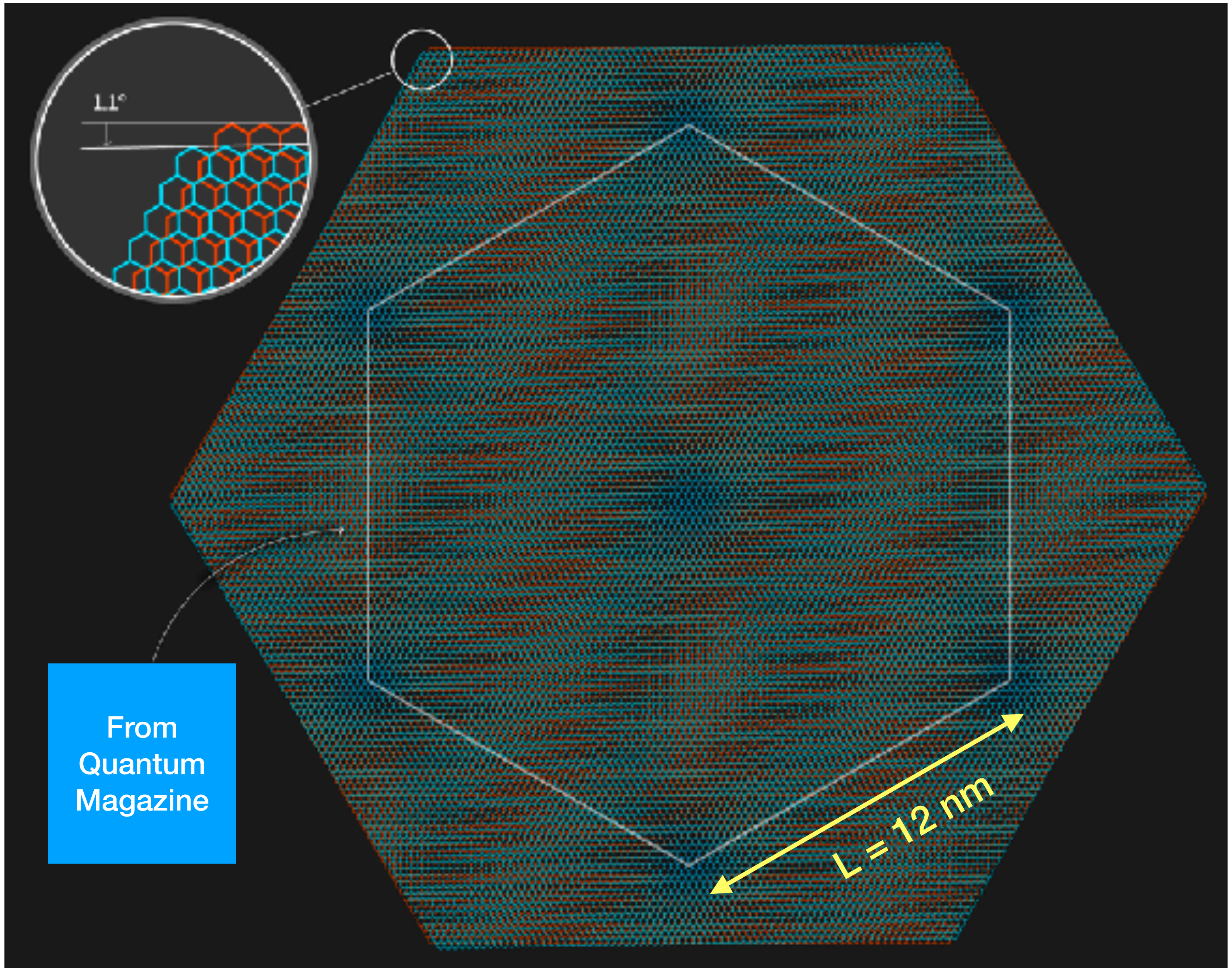


MAGIC ANGLE GRAPHENE



**MAGIC ANGLE ~1.1°:
Tunneling time =
Lattice Moire
time**

$$\theta \sim 1/60 \text{ radians} \qquad L \sim a/\theta$$



From Quantum Magazine

Continuum model

- Larger unit cell→ smaller BZone
- Bistrizer-Macdonald (BM) model (2011)

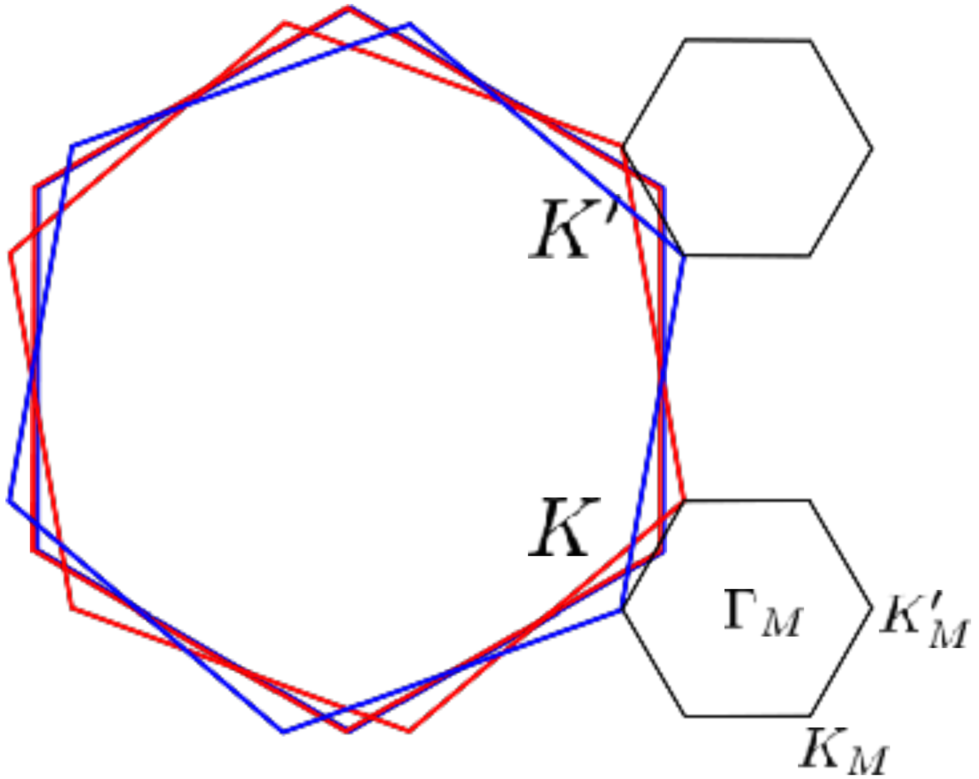
$$\mathcal{H}_K = \begin{pmatrix} -iv_F \boldsymbol{\sigma}_{\theta/2} \cdot \nabla & T(\boldsymbol{r}) \\ T^\dagger(\boldsymbol{r}) & -iv_F \boldsymbol{\sigma}_{-\theta/2} \cdot \nabla \end{pmatrix}_{12},$$

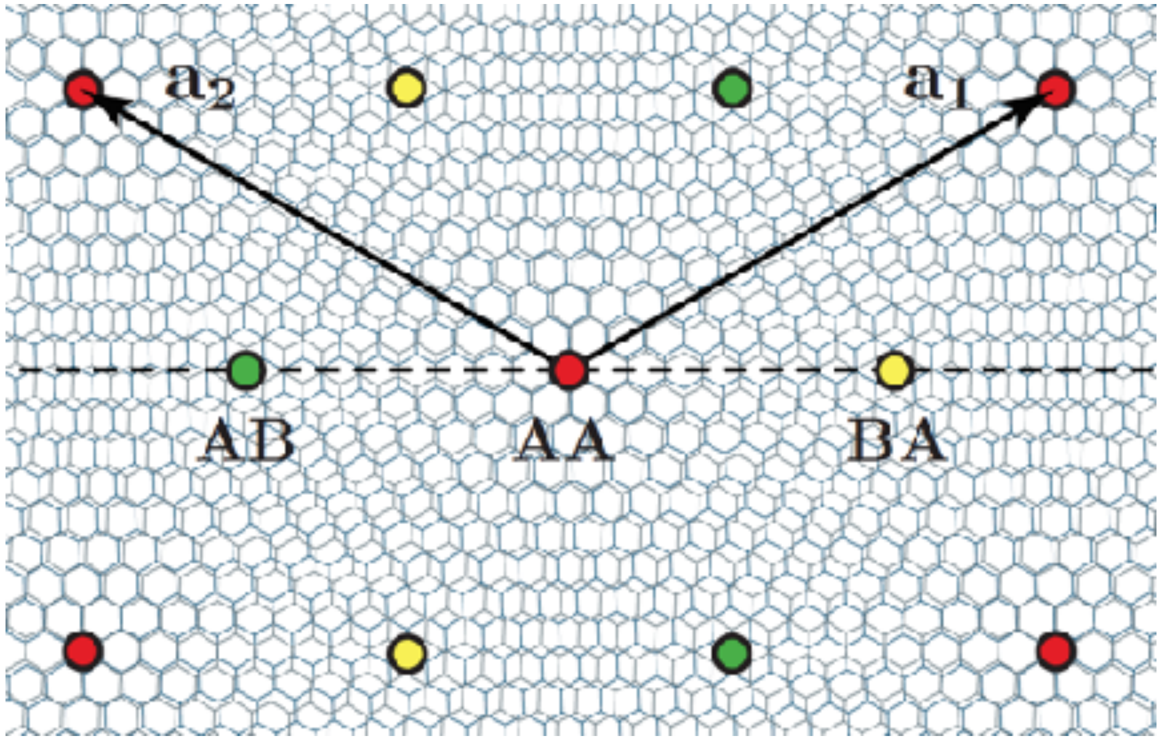
- Moire “potential”

$$T(\boldsymbol{r}) = \begin{pmatrix} w_0 U_0(\boldsymbol{r}) & w_1 U(\boldsymbol{r}) \\ w_1 U^*(-\boldsymbol{r}) & w_0 U_0(\boldsymbol{r}) \end{pmatrix}_{AB}$$

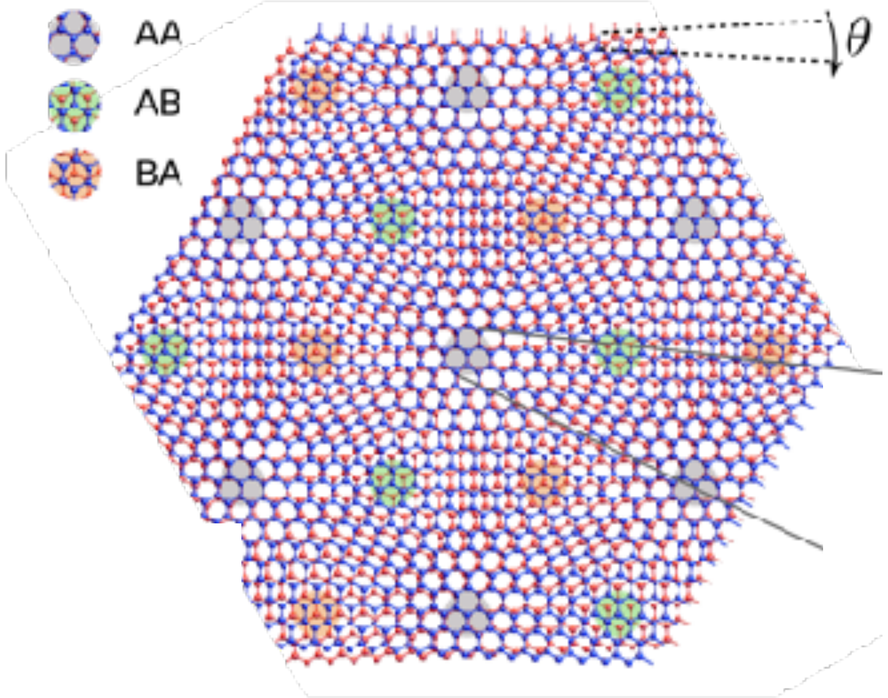
- Lattice relaxation: AB stacking favored to AA stacking (Carr *et al*. 2019, Nam, Koshino 2017)

$$\Longrightarrow \; w_0/w_1 \approx 0.7$$





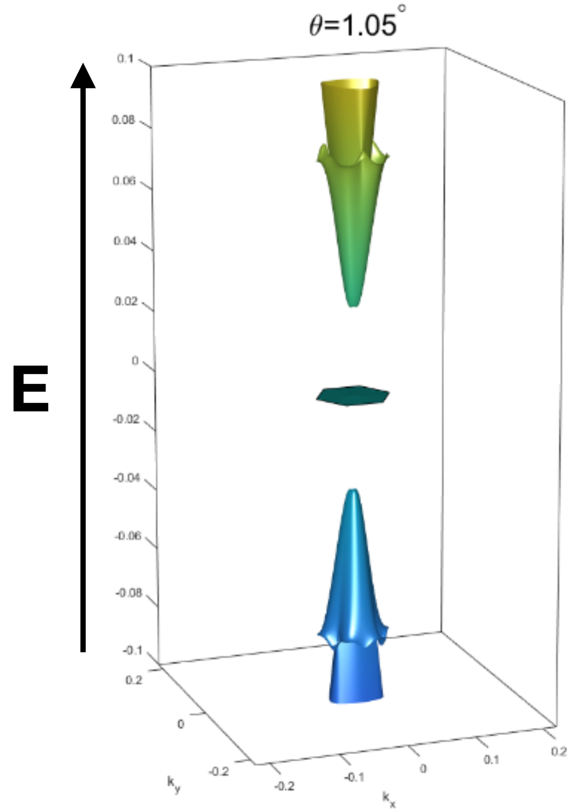
MAGIC ANGLE GRAPHENE

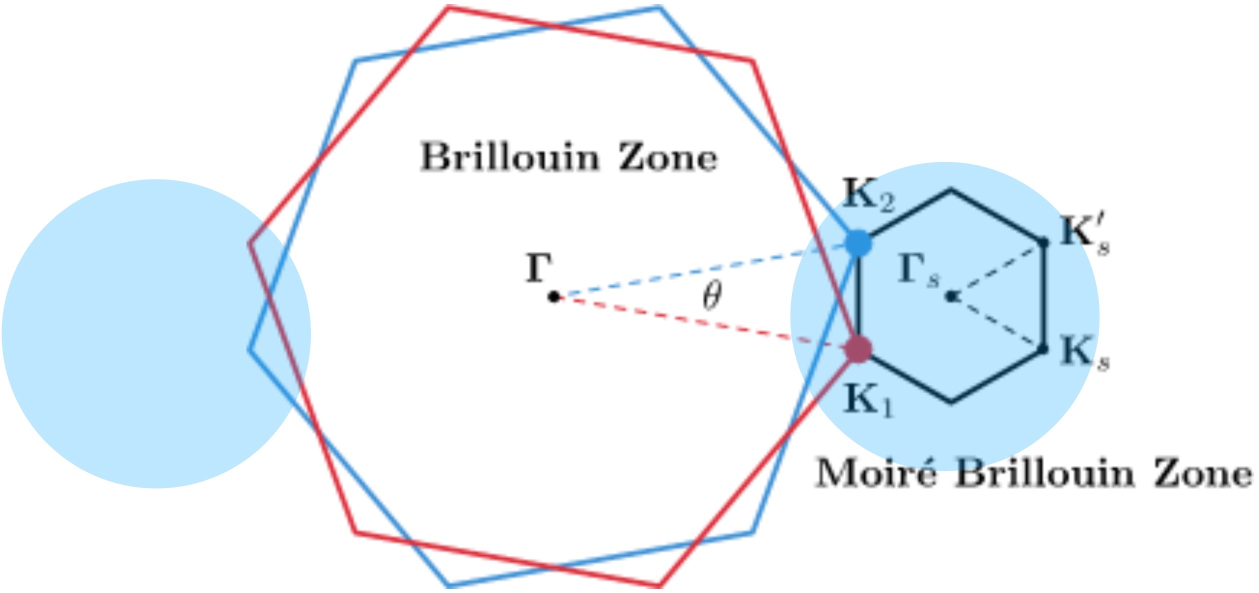


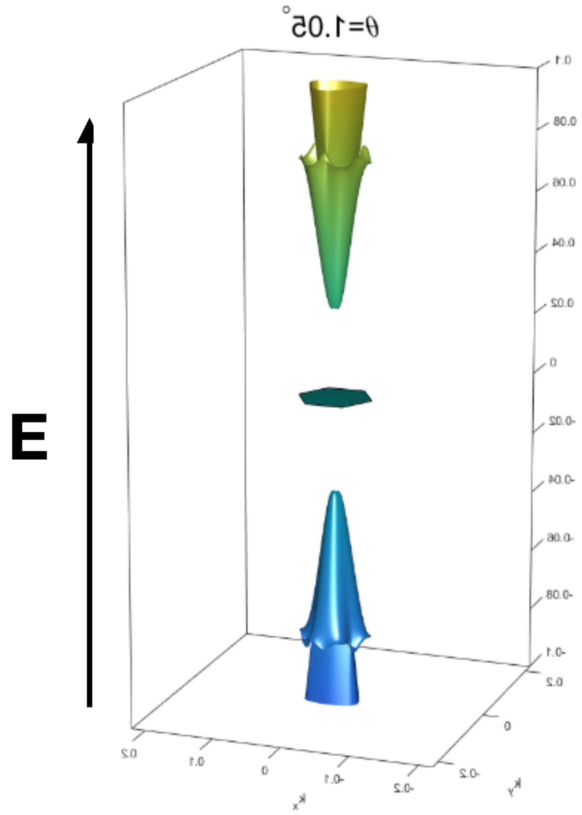
$$T(\mathbf{r}) = \begin{array}{cc} \begin{array}{cc} \quad A \quad & \quad B \quad \end{array} & \\ \begin{pmatrix} w_0U_0(\mathbf{r}) & w_1U_1(\mathbf{r}) \\ w_1U^*(-\mathbf{r}) & w_0U_0(\mathbf{r}) \end{pmatrix} & \begin{array}{c} A' \\ B' \end{array} \end{array}$$

Valley K

Valley K’



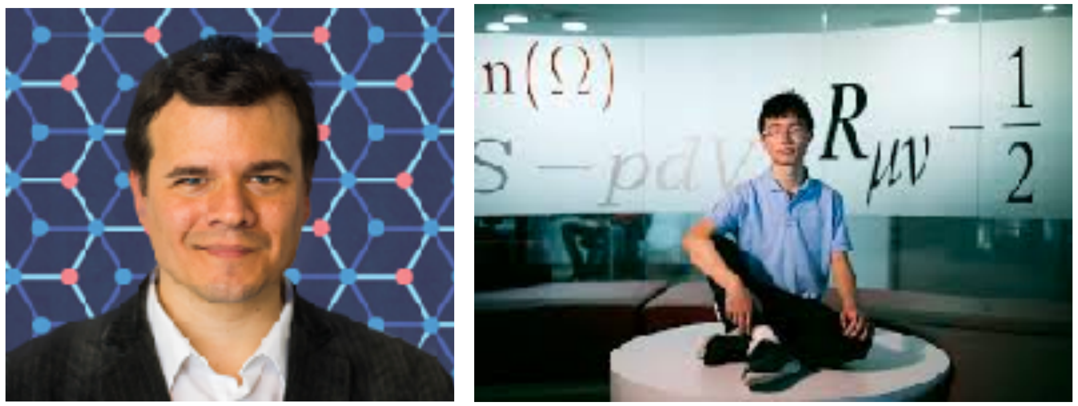




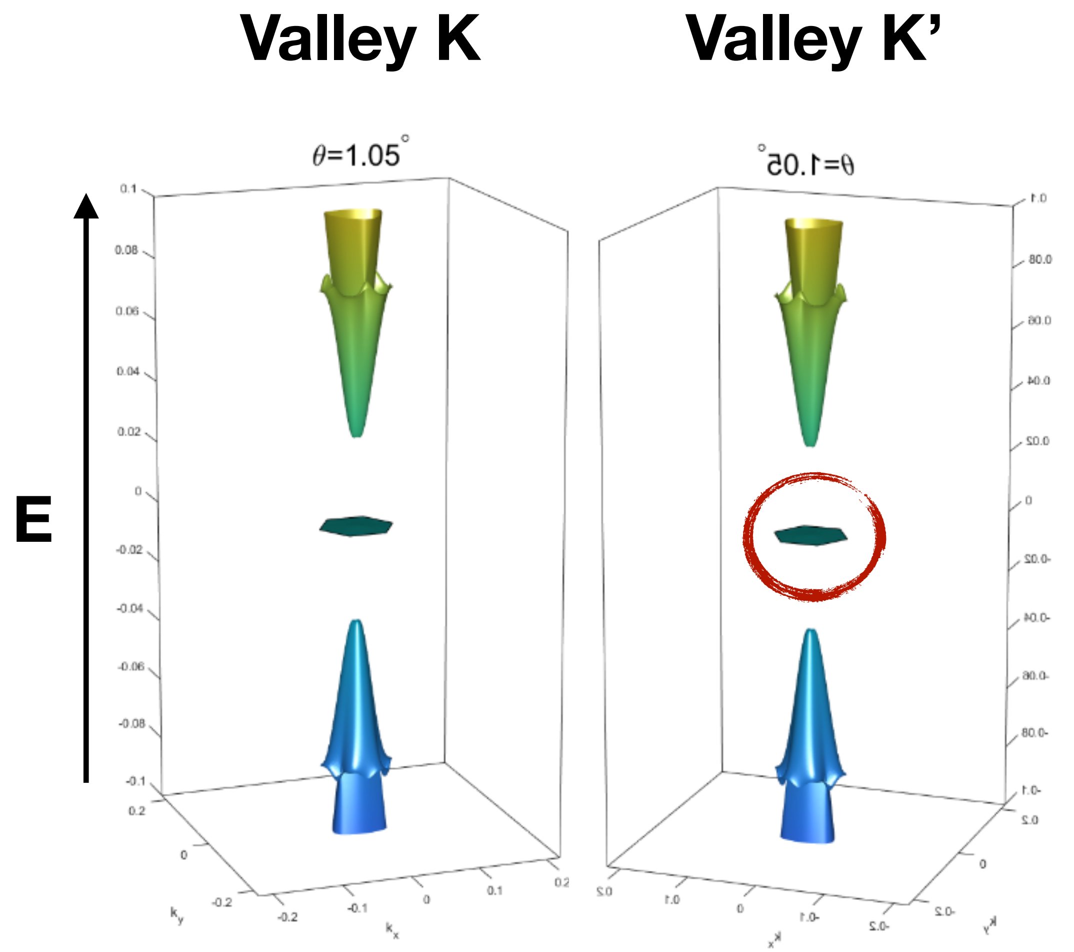
Momentum **Space**

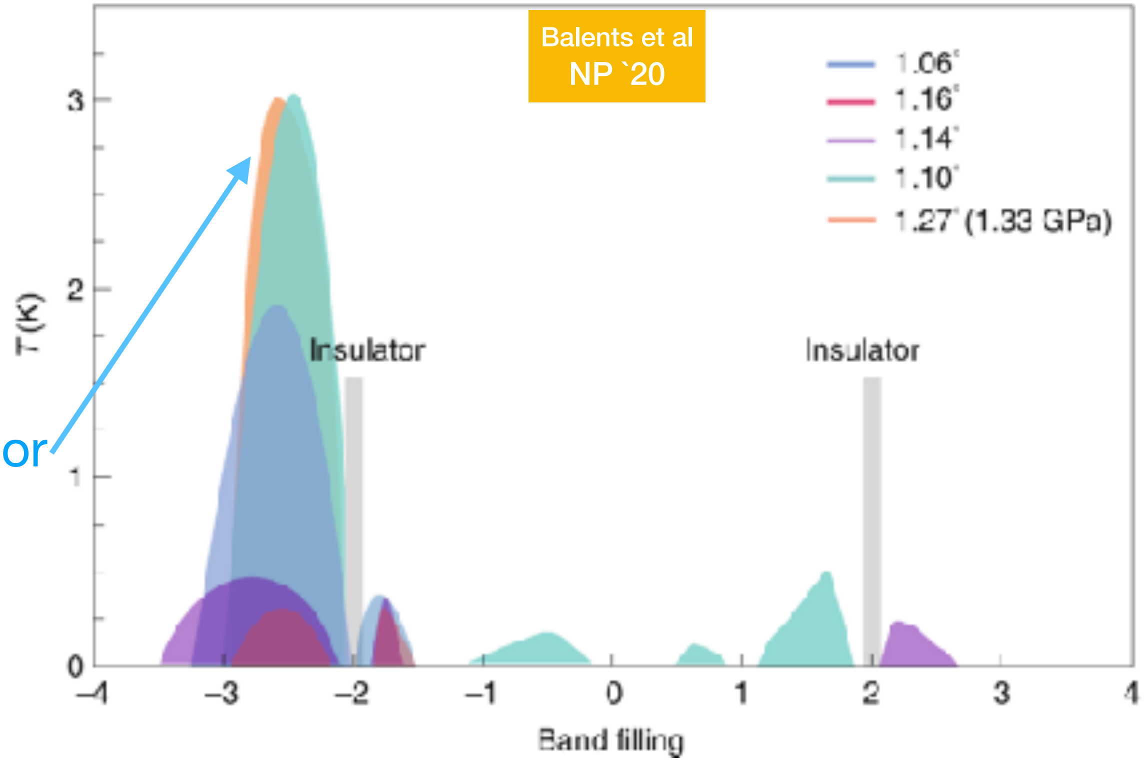
Y. Cao… (2018)

Experiments

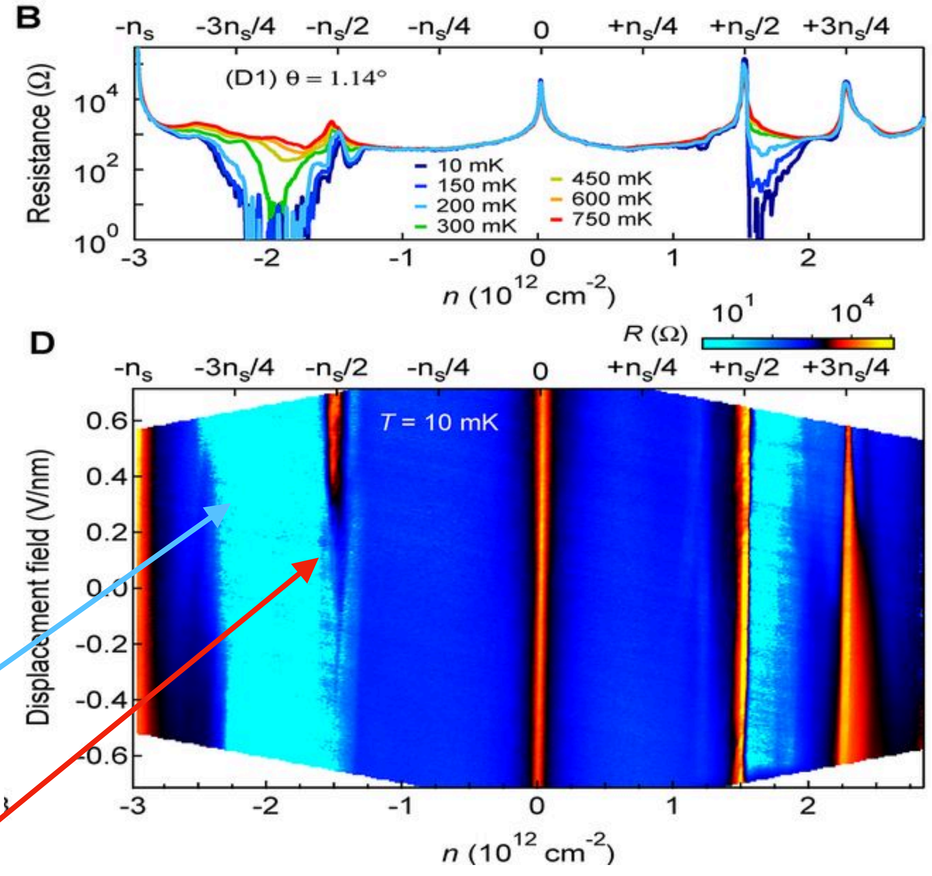


Pablo Jarillo-Herrero's group (MIT)
Cao *et al.* Nature 556, 80 (2018)
Cao *et al.* Nature 556, 43 (2018)





Superconductor



Superconductor

Insulators

Yankowitz et al. Science

Chiral Model

Tarnopolski, Kruchkov, AV
PRL 2019

$$T(\boldsymbol{r}) = \begin{pmatrix} \xcancel{w_0 U_0}(\boldsymbol{r}) & w_1 U(\boldsymbol{r}) \\ w_1 U^*(-\boldsymbol{r}) & \xcancel{w_0 U_0}(\boldsymbol{r}) \end{pmatrix}$$

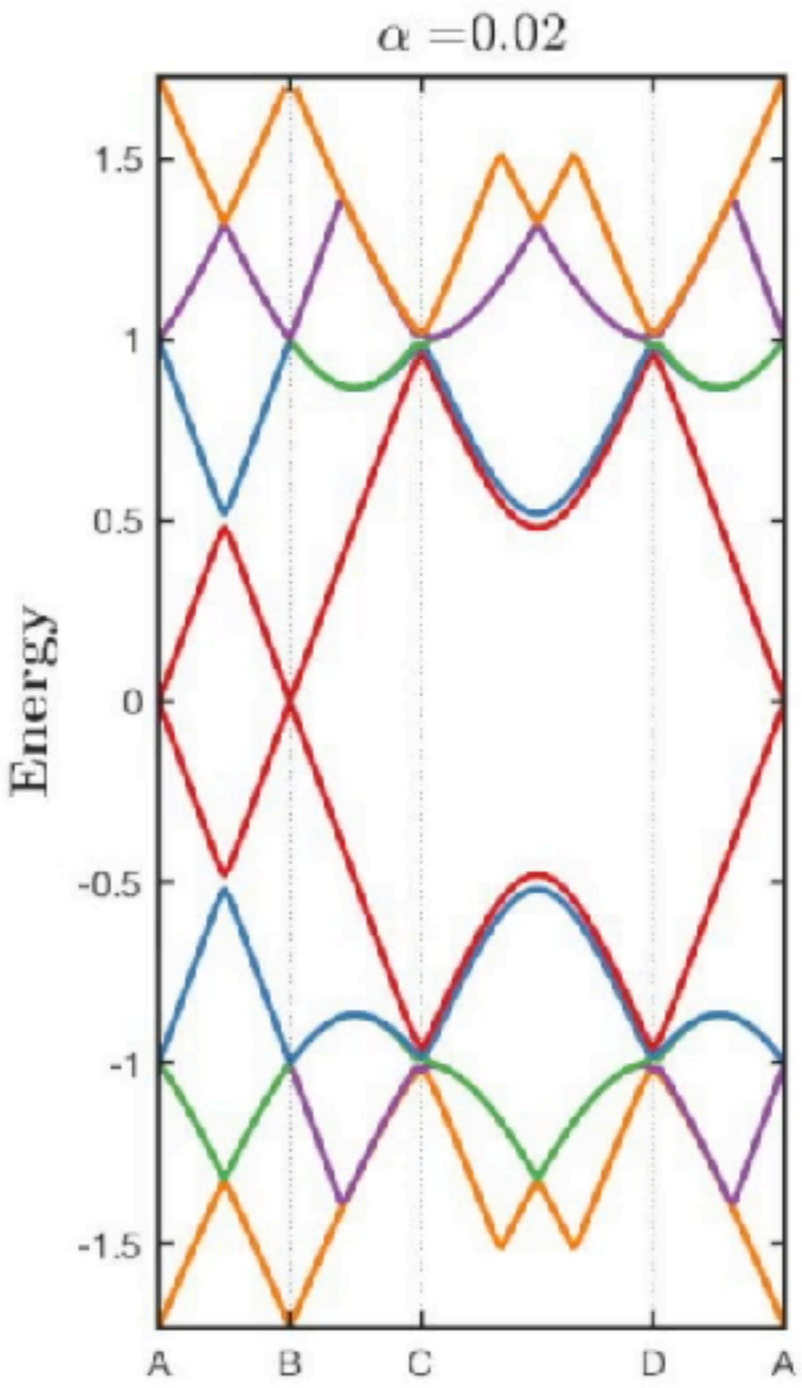
Only AB coupling

Chiral Symmetry

$$\{\sigma_z \otimes 1, \mathcal{H}\} = 0$$

sublattice

Perfectly Flat Bands in the Chiral Model



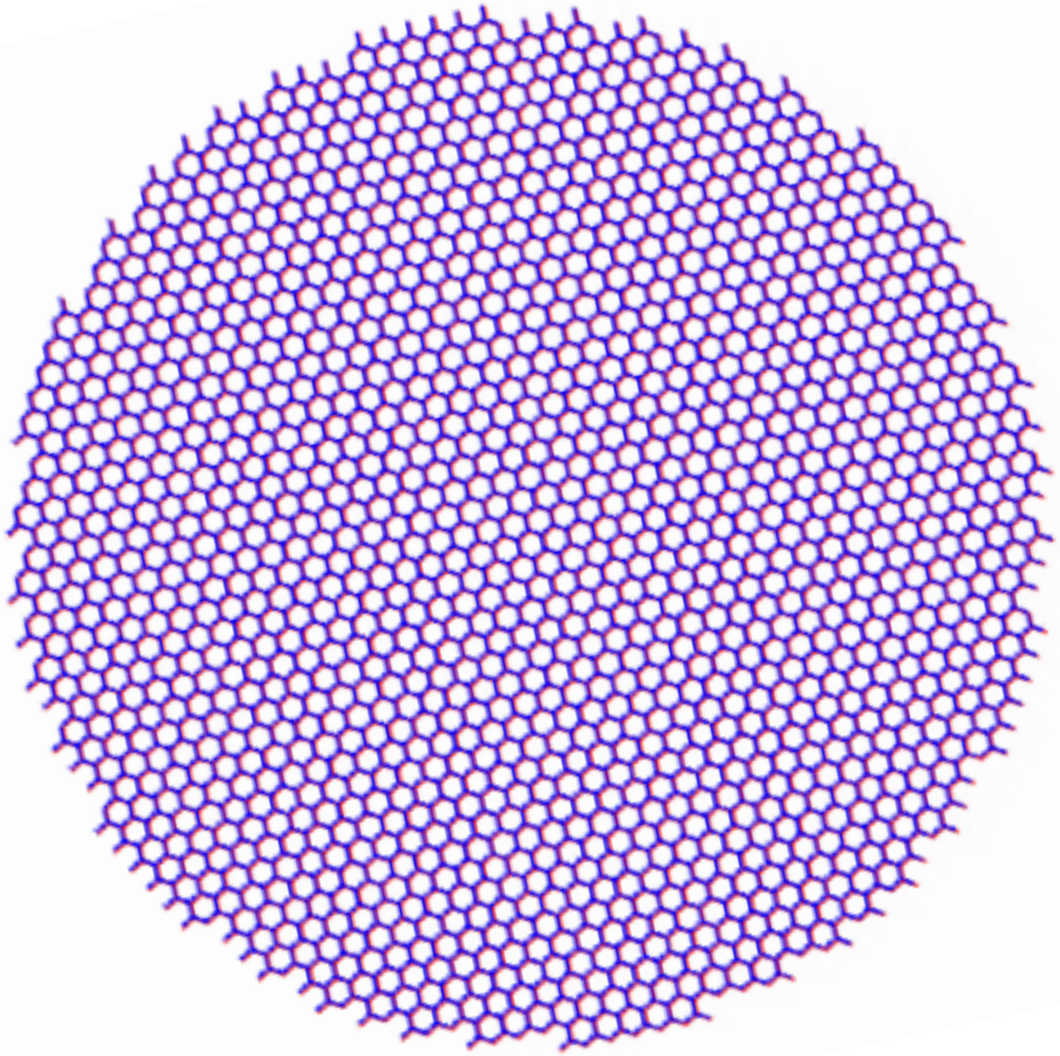
$$\alpha = \frac{0.6}{\theta^o}$$

Graphene’s Electrons in Moiré lattice

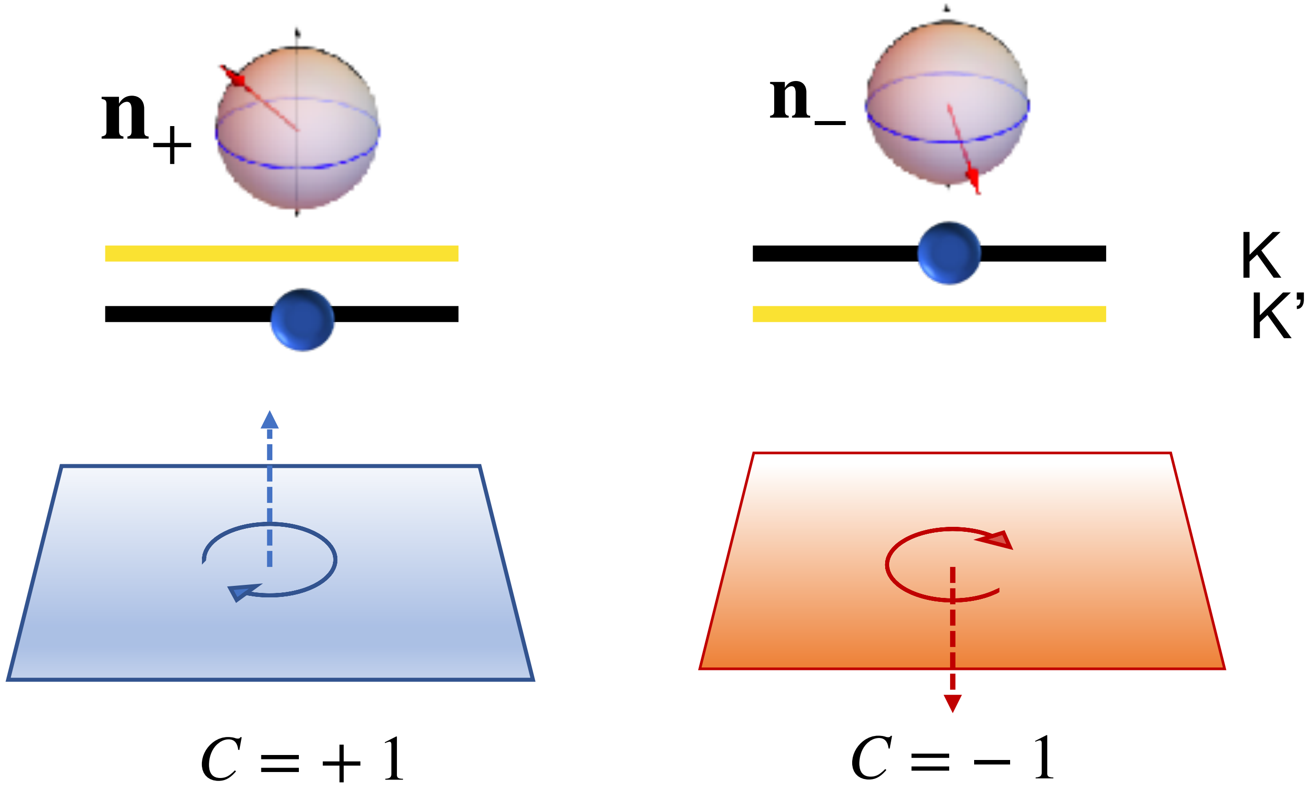
Graphene Electrons

\+ Moire lattice

\+ INTERACTIONS



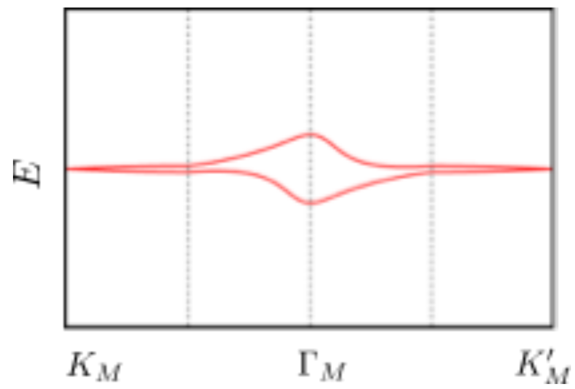
Insulators

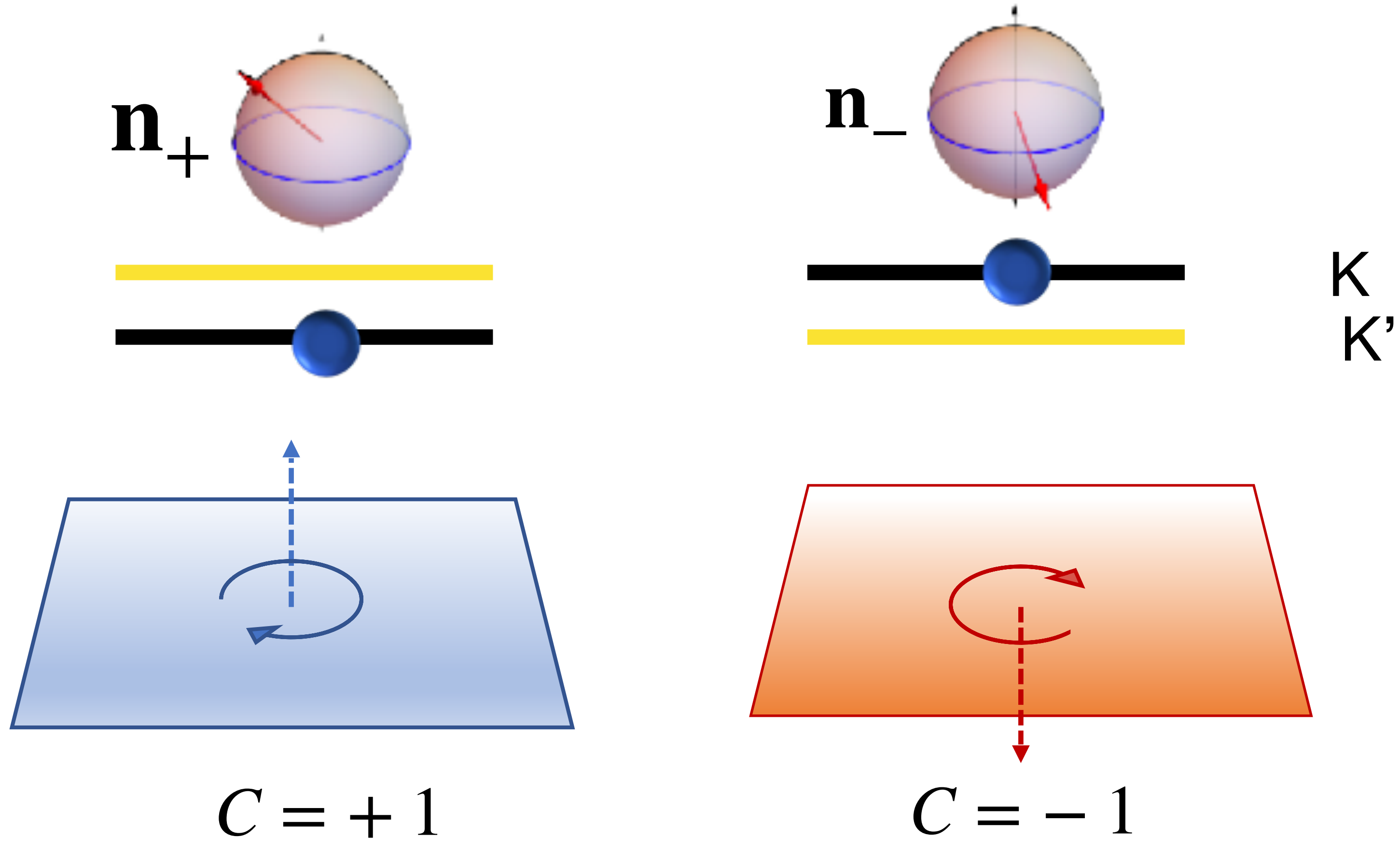


$\mathbf{n}_+ \ \mathbf{n}_-$

- Fill Bands. - requires flavor (spin/valley) ordering
- σ Model description - $J n_+ \cdot n_-$

Landau Level Picture of Flat bands



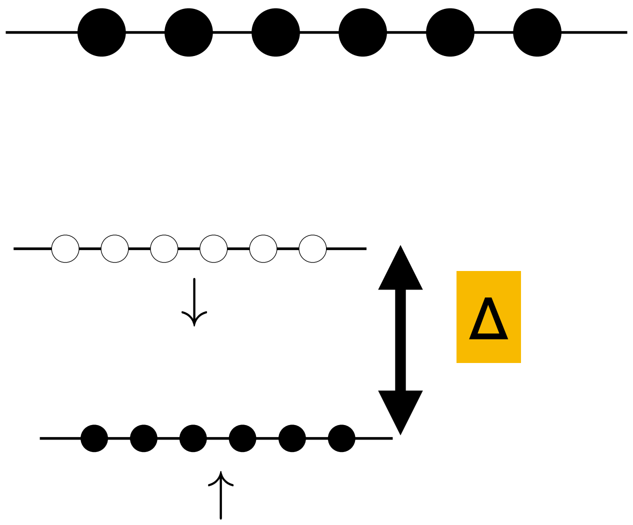


$\mathbf{n}_+$ $\mathbf{n}_-$

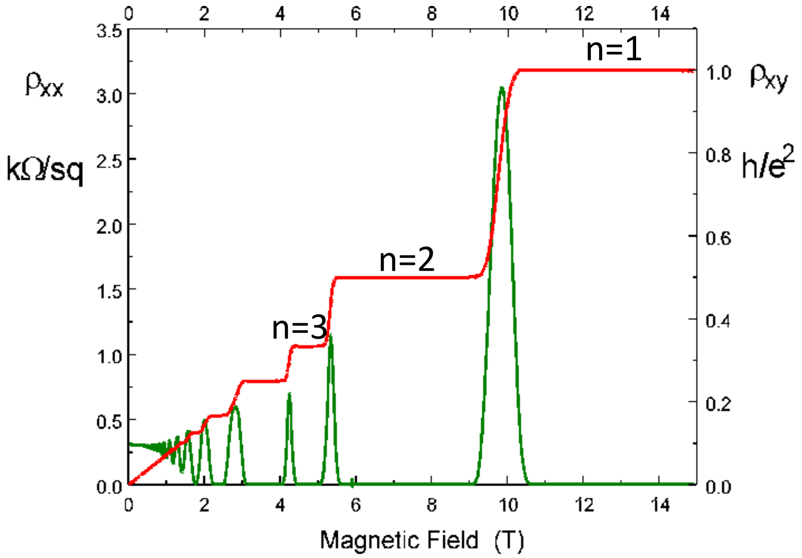
- Fill 2 out of 4 bands.
- σ Model - but topological features.

San Jose et al PRL (2012), Tarnopolsky *et al*, PRL (2019), J. Liu *et al*, PRB (2019)

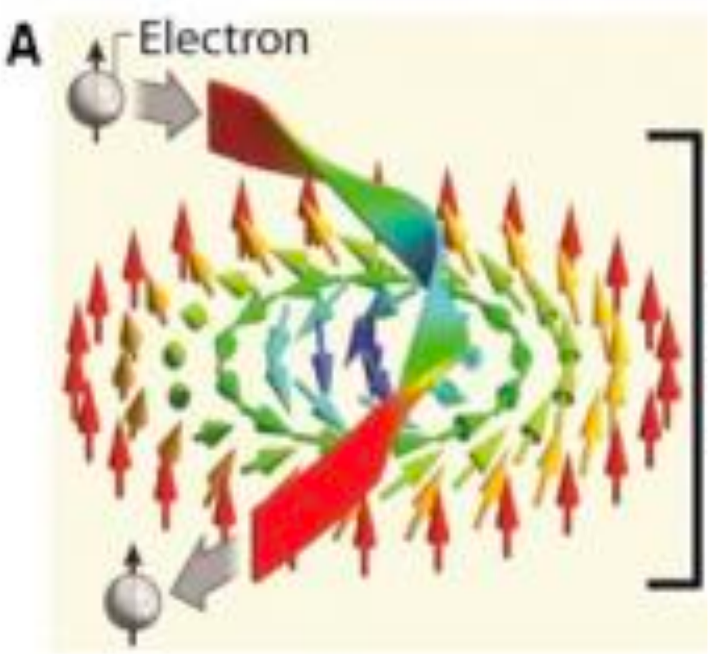
Quantum Hall Ferromagnet and Skyrmions



A spin-degenerate Landau level



- Anything interesting combining symmetry breaking + band topology?

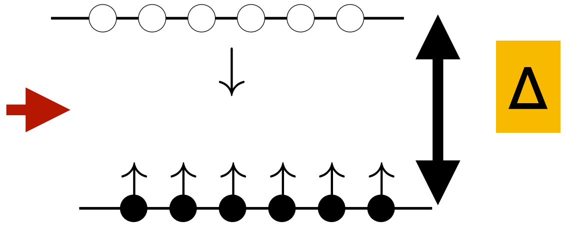


Electric charge carried by *Skyrmions*.

Quantum Hall Ferromagnet and Skyrmions

E$_{\mathbf{Skx}}$

$=8\pi\rho$

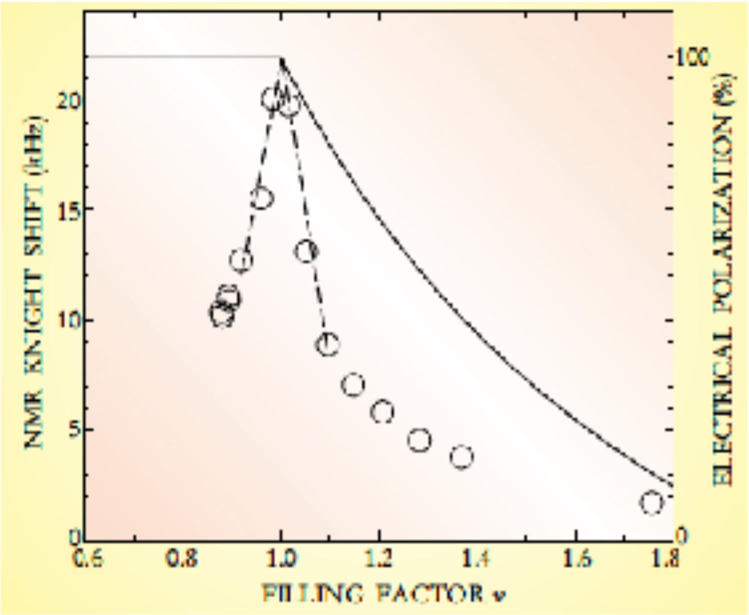


A spin-degenerate Landau level

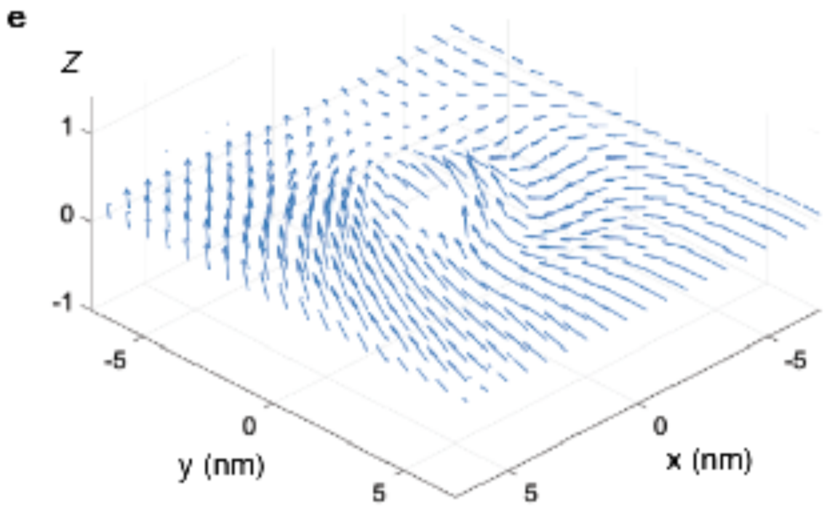
- Quantum Hall Ferromagnet

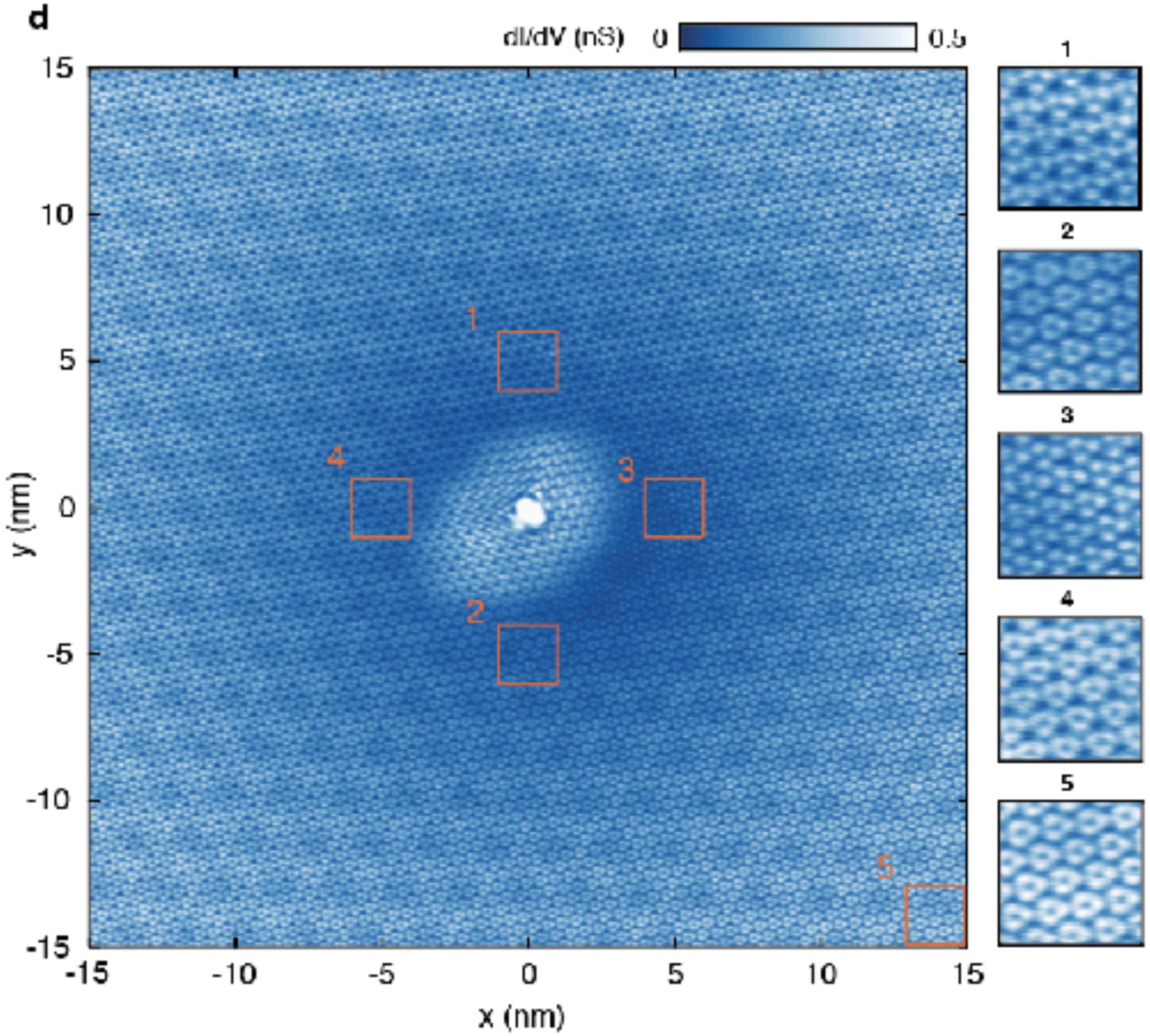
Skyrmions are **Charged**

Cheapest charge excitation is the *Skyrmion*

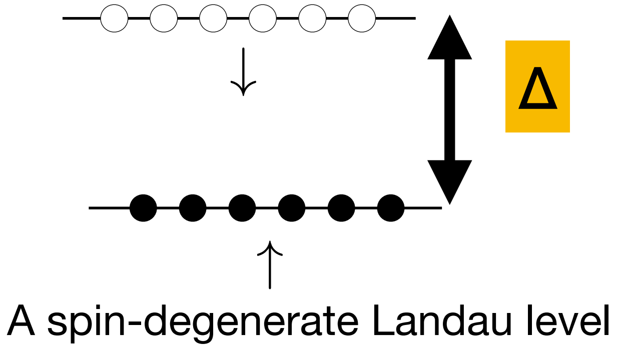


Theory: Sondhi, Karlhede, Kivelson, Rezayi; Lee & Kane
Experiments: NMR - Barrett et al. Liu…Zaletel, Yazdani.





What is Fundamental?



A spin-degenerate Landau level

Electron -> ferromagnet

Ferromagnet -> electron

Tony Skyrme

A UNIFIED FIELD THEORY OF MESONS AND BARYONS

T. H. R. SKYRME[†]
A.E.R.E., Harwell, England

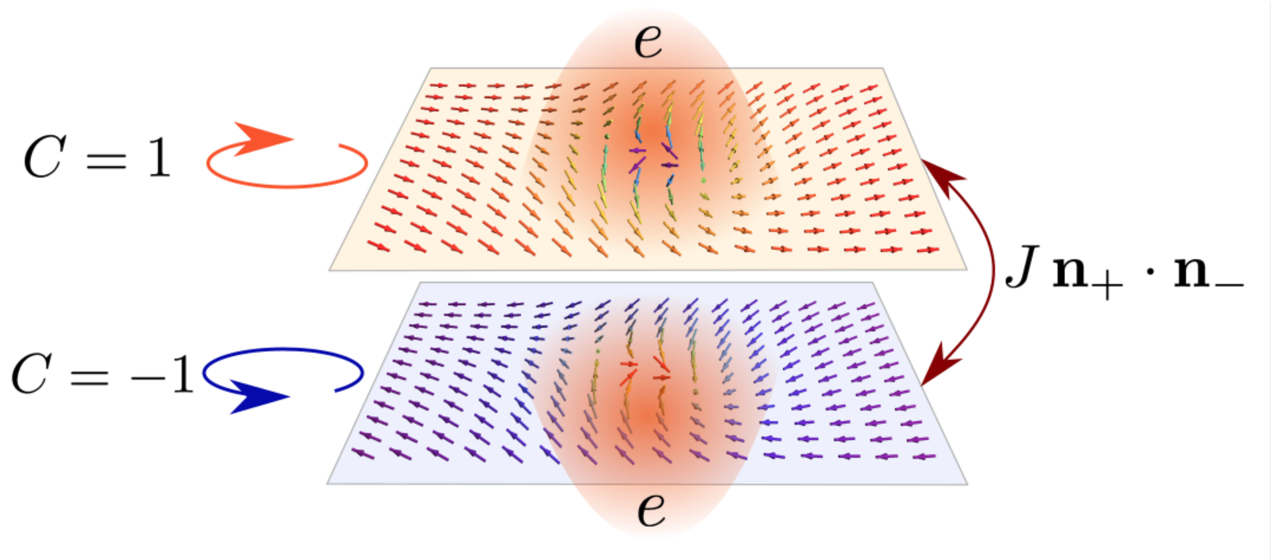
Nuclear Physics **31** (1962) 556—569;

[†] Now at Department of Mathematics, University of Malaya, Pantai Valley, Kuala Lumpur, Malaya.

We are indebted to Mr. A. J. Leggatt for carrying out the calculations reported in sect. 3, while a vacation student at A.E.R.E.

Charged Skyrmions

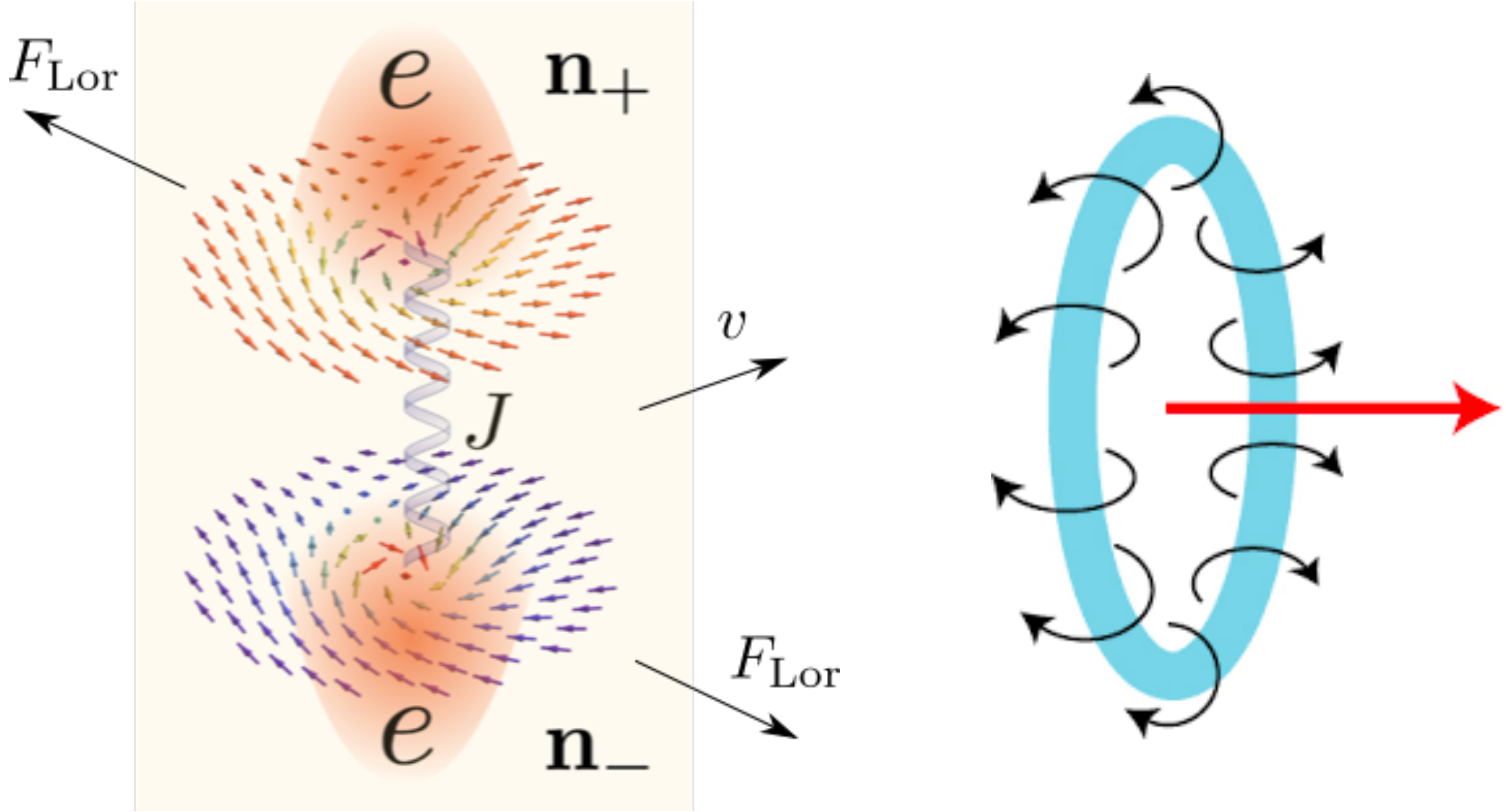
- Charge $2e$ bound state!



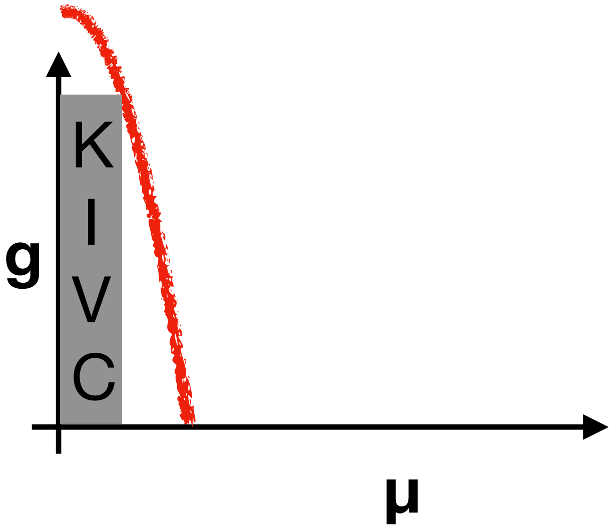
EK, Chatterjee, Bultinck, Zaletel, Vishwanath arxiv:2004.00638

Effective Mass

$$T_c = \frac{\nu\pi\hbar^2}{2A_M M_{\text{pair}}} = \nu\frac{JA_M}{2}$$



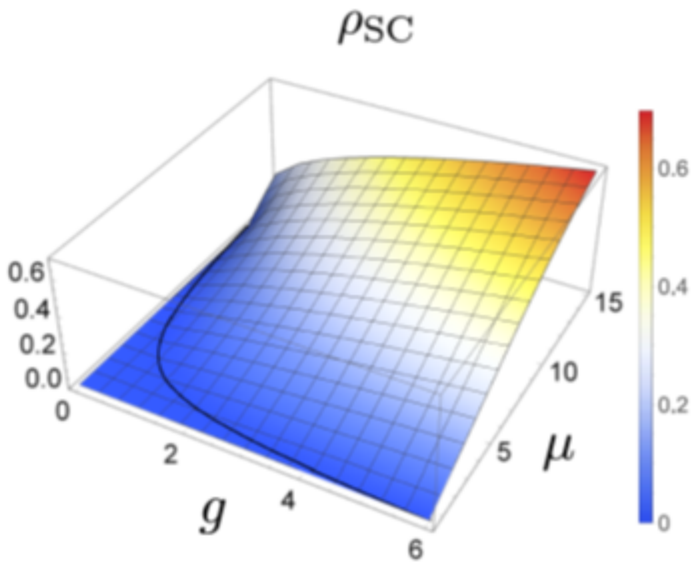
Phase Diagram & Stiffness

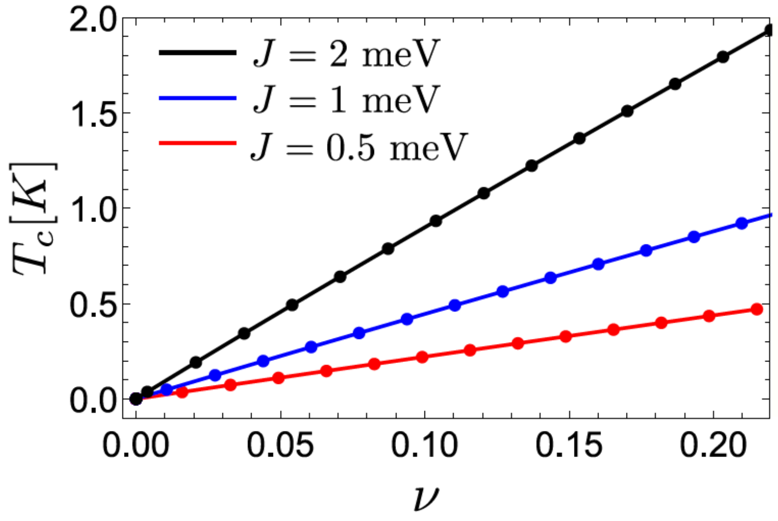


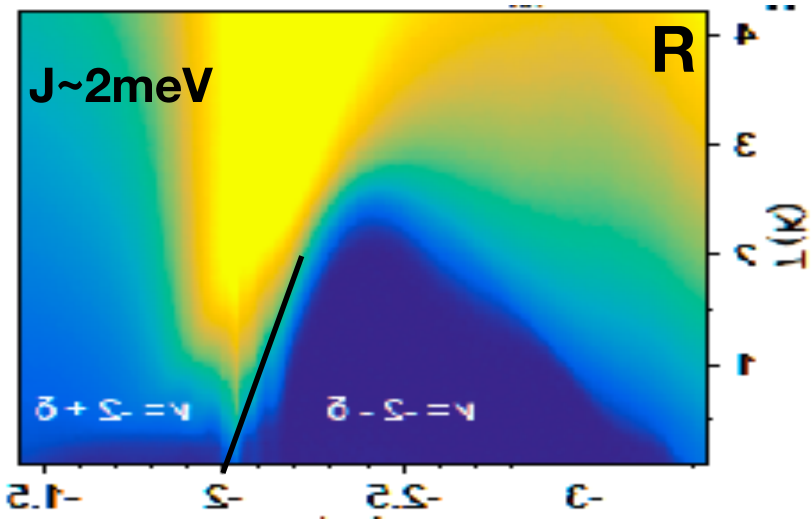
Sigma model -
ORDERED- Insulator
DISORDERED - Superconductor

Solve using large-N CP$_N$

$$\mathcal{L} = \frac{\Lambda}{g}|(\partial_\mu - i a_\mu) z|^2 + \quad \mu \frac{b}{\pi}$$







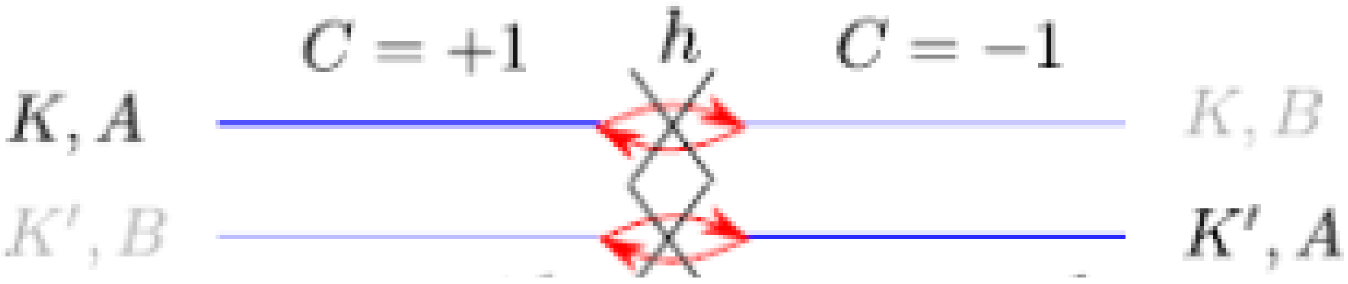
J. Park, Y. Cao … Pablo `20

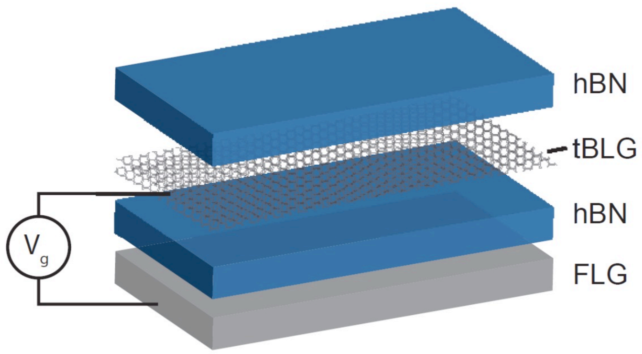
EK, Chatterjee, Bultinck, Zaletel, Vishwanath arXiv:2004.00638

Chatterjee *et al.* arxiv:2010.01144

Essential Ingredients for Superconductivity?

- $C_2\mathscr{T}$ symmetry helps superconductivity in this setup
- Sublattice potential *suppresses* the coupling J





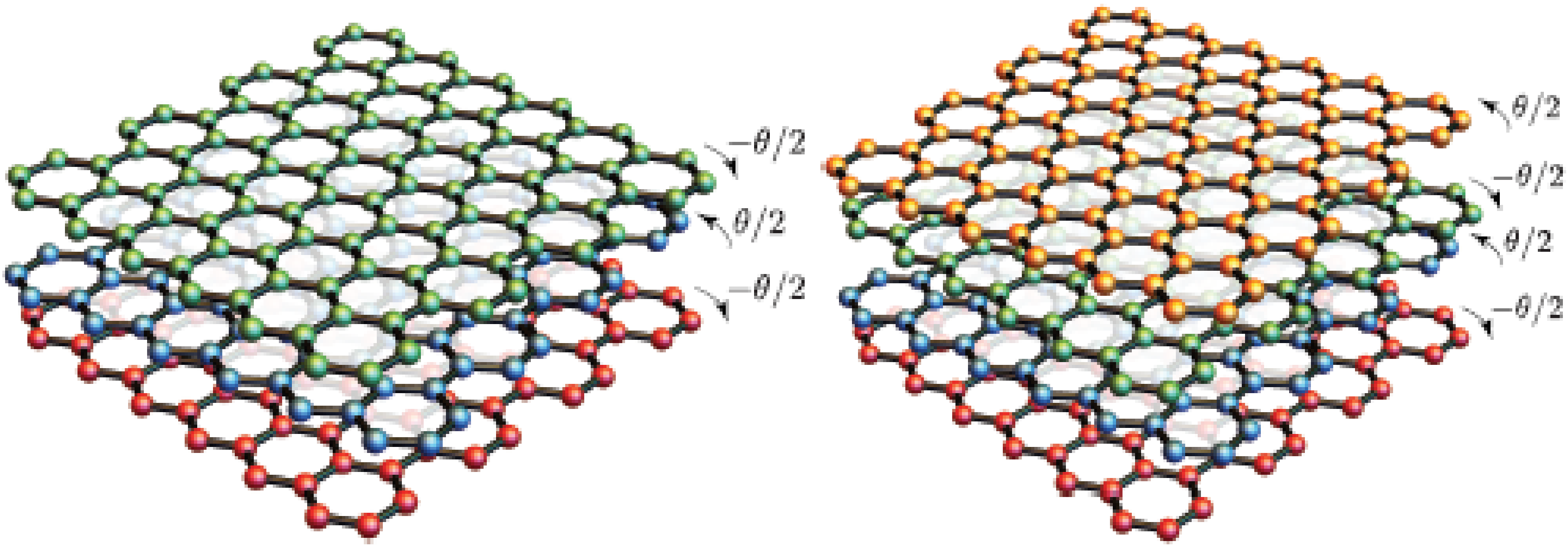
- Twisted bilayer graphene aligned on hBN → no superconductivity (Sharpe et al. Science 19, Serlin et al. Science 20)

- Relatively *few* Moire materials apart from magic angle graphene with this symmetry.

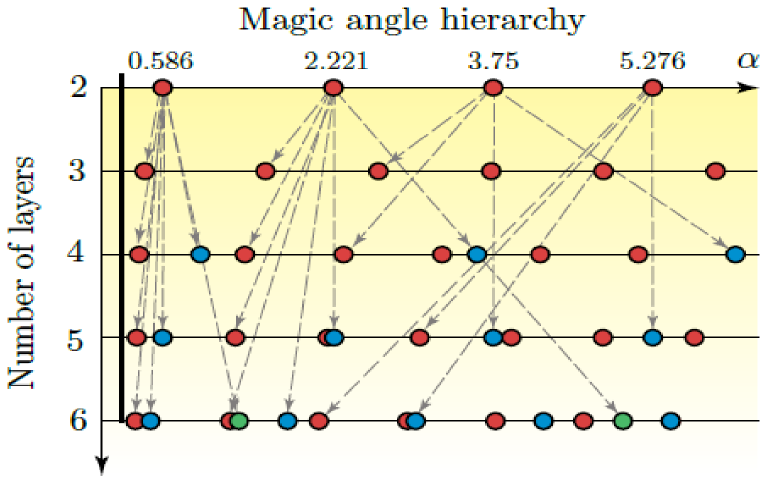
New platforms for Superconductivity?

- Other systems with C2 symmetry

Alternating twist sandwich



Eslam Khalaf

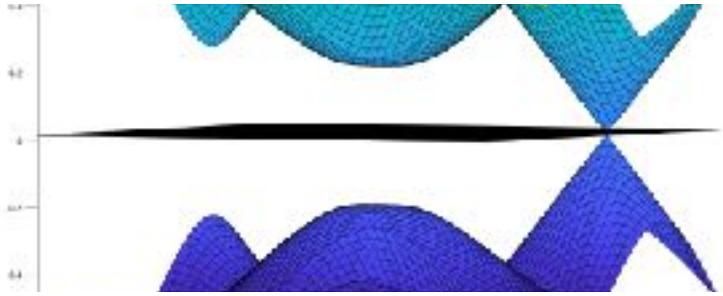


$\sqrt{2}$

$$\varphi = \frac{1+\sqrt{5}}{2}$$

Larger magic angles!

Magic angle = $\sqrt{2}\, 1.1^o \sim 1.55^o$



Flat band+Dirac

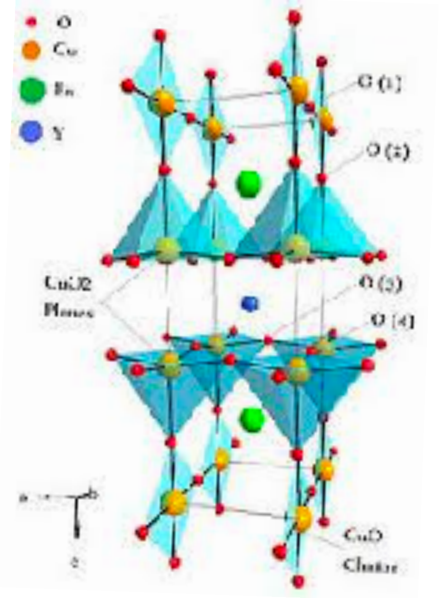
Carr et al. stability `19

Khalaf, Kruchkov, Tarnopolsky, AV, 1901.10485

Matthias' Rules for Superconductivity

Bernd Matthias (1918-1980)

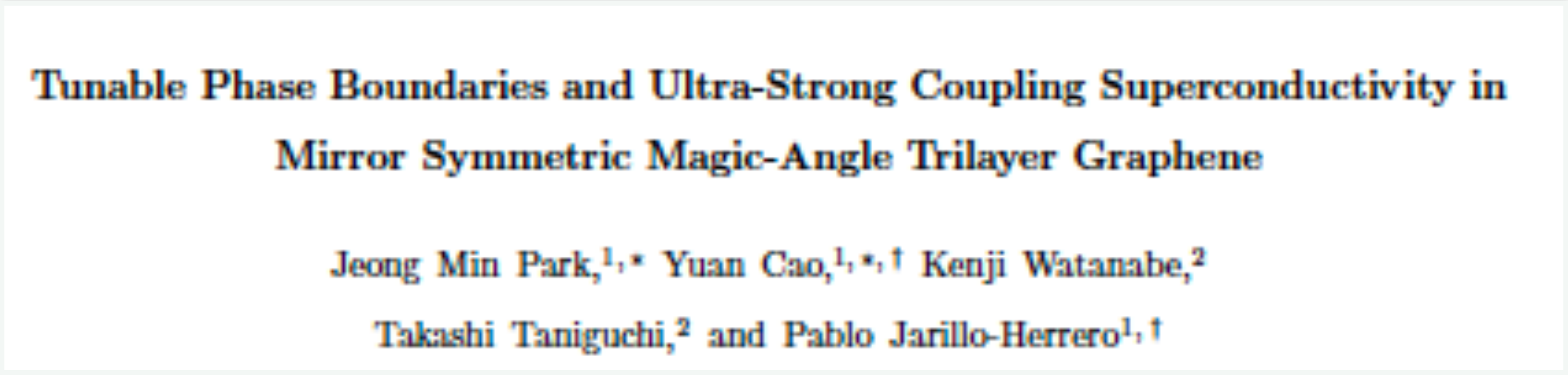
1. high symmetry is good, cubic symmetry is the best
2. high density of electronic states is good
3. stay away from oxygen
4. stay away from magnetism
5. stay away from insulators
6. stay away from theorists.



PABLO H-J

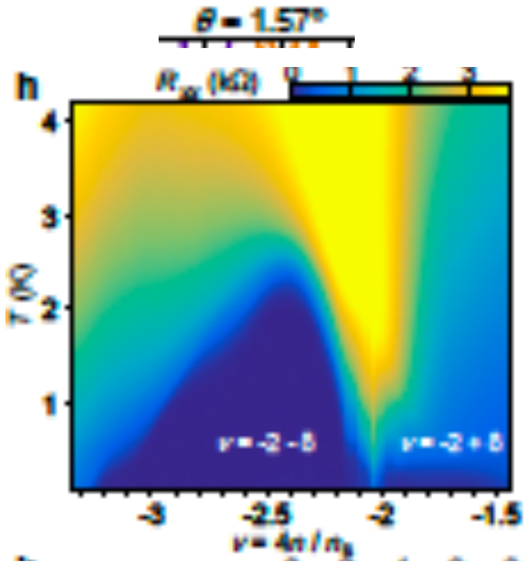
PHILIP KIM

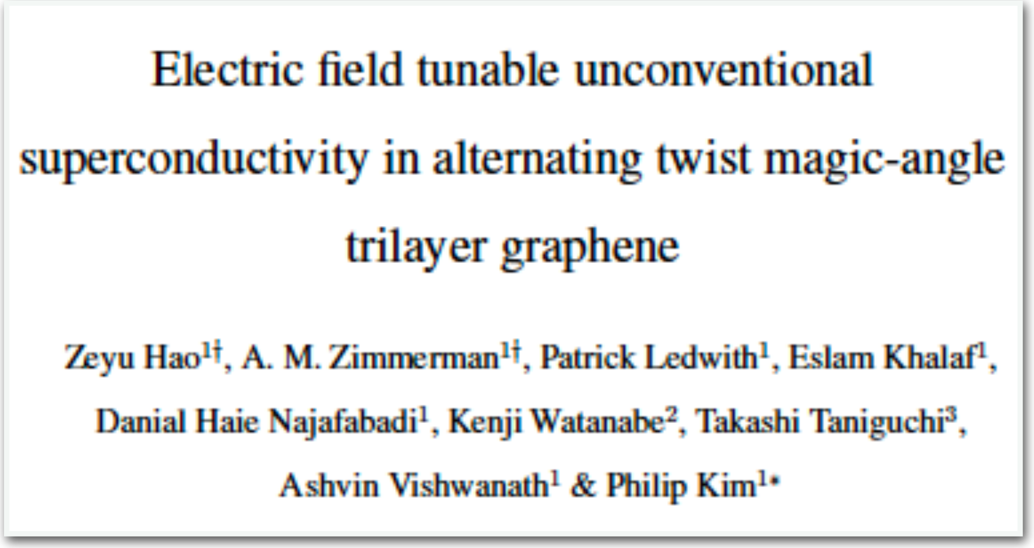
Experiments on Alternating Twist Trilayer



Tunable Phase Boundaries and Ultra-Strong Coupling Superconductivity in Mirror Symmetric Magic-Angle Trilayer Graphene

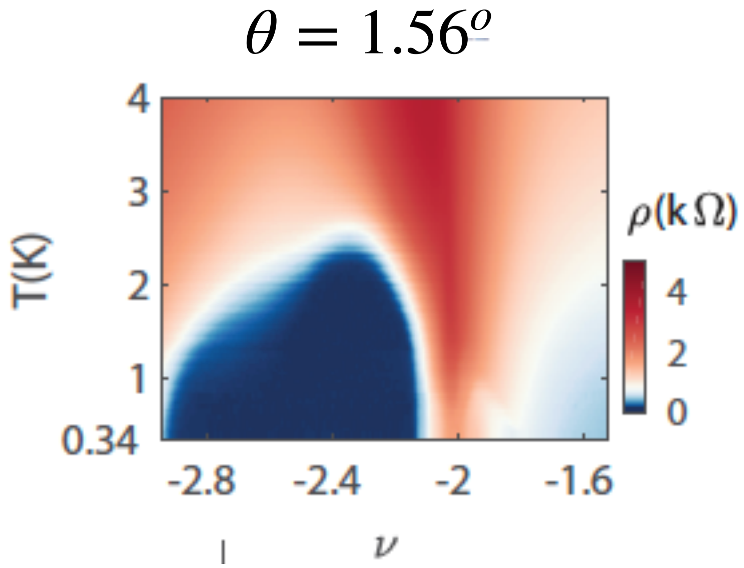
Jeong Min Park,[1,*] Yuan Cao,[1,*,†] Kenji Watanabe,[2]
Takashi Taniguchi,[2] and Pablo Jarillo-Herrero[1,†]

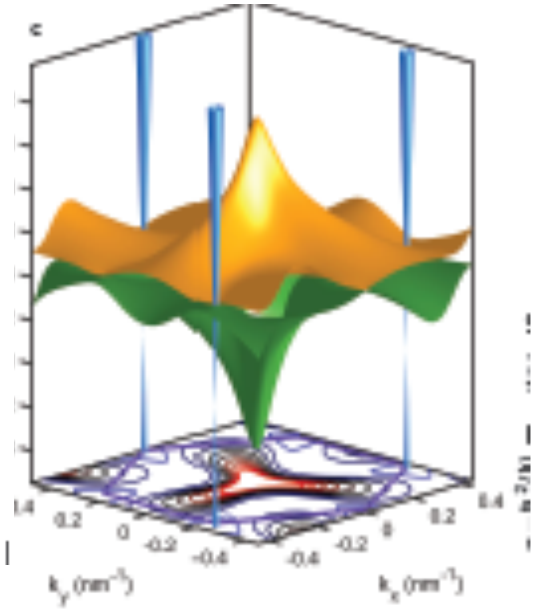




Electric field tunable unconventional superconductivity in alternating twist magic-angle trilayer graphene

Zeyu Hao[1‡], A. M. Zimmerman[1‡], Patrick Ledwith[1], Eslam Khalaf[1],
Danial Haie Najafabadi[1], Kenji Watanabe[2], Takashi Taniguchi[3],
Ashvin Vishwanath[1] & Philip Kim[1*]





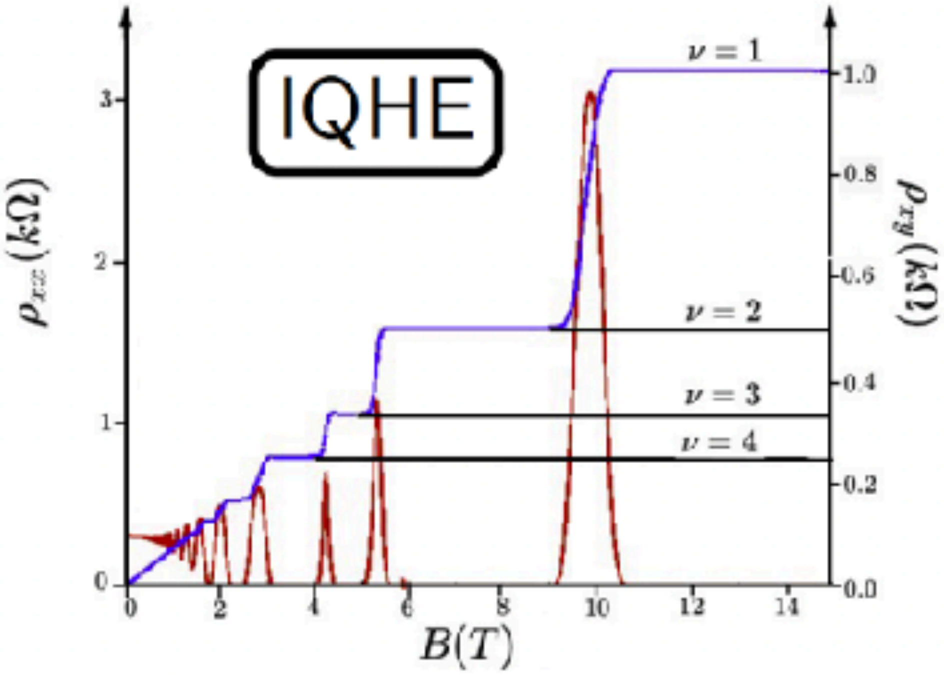
+ many others now. Also 4 layers (at golden mean) and 5 layers also shown to be superconducting

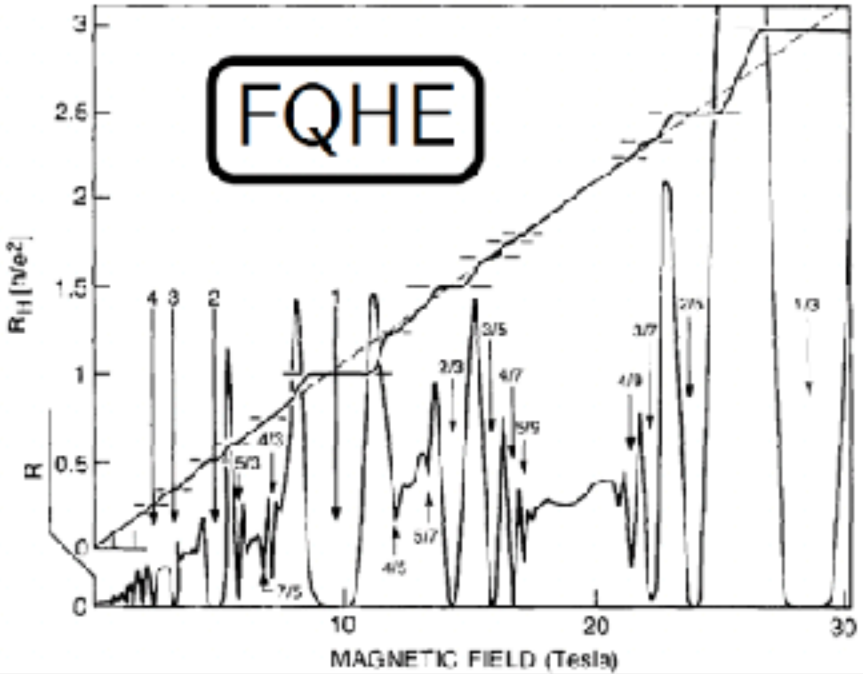
Chern Insulators and Fractional Chern Insulators in Magic Angle Graphene

Integer Charge

Fractional Charge

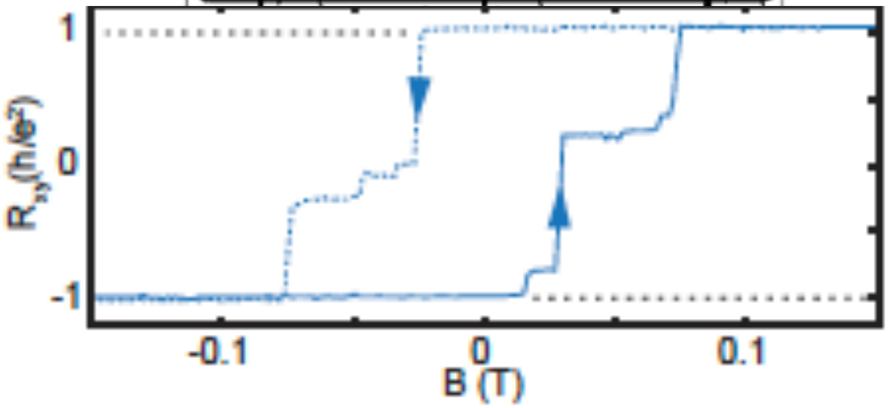
(*B* Field)





Band Topology/ Geometry

Chern Insulator



Fractional Chern Insulator

FCIs

by fractional filling of Chern band?

Figures: Wikipedia; Stormer (1992);

Magic angle graphene (+ aligned hBN)

Serlin et al. Sharpe et al. Science `19. Stepanov et al. PRL `21

Intrinsic Topological Order

Conclusions & Future Directions

- Skyrmions
    - A soluble strong coupling theory - not local DOS related.
    - associated with nu=2
    - Connection to C_2T symmetry, accounts for robust Sc platforms.
    - Predicts new platforms.
- Future Directions & Challenges
    - We want to incorporate fermions into the theory (coexisting fermions + Skx).
    - Connection to weak coupling theories?
    - Role of strain in magic angle graphene?

Lecture Notes:**https://scholar.harvard.edu/avishwanath/teaching**

Collaborators

Eslam Khalaf

Harvard

Nick Bultinck

Shubhayu Chatterjee

Berkeley

Mike Zaletel

Shang Liu

Harvard

Patrick Ledwith

Harvard

Theory: Jong Yeon Lee, Daniel Parker, G. Tarnopolsky, A. Kruchkov, Adrian Po, T. Senthil, Liujun Zou

Experiment: Kim group, Yacoby group, Yazdani group.

Discussions: Pablo Jarillo Herrero, Yuan Cao, Jane Park, Andrea Young, Allan MacDonald