# Topics for the Qualifying Examination

## Quantum Mechanics I and II

1. Quantum kinematics and dynamics
   1.1 Postulates of Quantum Mechanics.
   1.2 Configuration space vs. Hilbert space, wave function vs. state vector
   1.3 Time evolution in Schrodinger and Heisenberg pictures.
2. Simple 1-d problems
   2.1 Free particle
   2.2 Delta function potential.
   2.3 Step-like potentials.
   2.4 Harmonic oscillator: eigenstates and spectrum, raising and lowering operators.
   2.5 Particle in a box.
   2.6 Uncertainty principle.
3. Continuous symmetries
   3.1 Translations.
   3.2 Rotations.
4. Discrete symmetries
   4.1 Parity
   4.2 Time reversal.
5. Angular momentum
   5.1 Orbital angular momentum.
   5.2 Spin.
   5.3 Angular momentum algebra, addition of angular momentum.
6. Spherical symmetry
   6.1 Spherical potentials.
   6.2 Spherical harmonics.
   6.3 Wigner-Eckart theorem.
   6.4 Example: hydrogen atom.
7. Charged particle in a magnetic field
   7.1 Electromagnetic gauge transformations.
   7.2 Landau levels.
   7.3 Aharanov-Bohm effect.
8. Identical particles: Fermi and Bose statistics.
9. Perturbation theory
   9.1 Time independent perturbation theory.
   9.2 Time dependent perturbation theory.
   9.3 Selection rules.
10. Other approximation methods
    10.1 Born approximation.

10.2 WKB approximation.
10.3 Sudden and adiabatic approximations.

10. Variational methods.
11. Scattering theory
    11.1 Partial wave expansion.
    11.2 Phase shifts.
    11.3 Resonant scattering.
12. Second quantization.
13. Dirac equation.

## Statistical Mechanics

1. Thermodynamics
    1.1 Thermodynamic equilibrium , First Law of thermodynamics
    1.2 Second Law of thermodynamics, Carnot cycle, absolute temperature, entropy
    1.3 Relations between derivatives of thermodynamic quantities.
    1.4 Thermodynamic potentials.
    1.5 Third Law of thermodynamics.
2. Classical Statistical Mechanics
    2.1 The postulates of classical statistical mechanics (equal *a priori* probabilities), Loiuville's theorem, and the ergodic hypothesis.
    2.2 Microconanonical ensemble and derivation of thermodynamics.
    2.3 Equipartition theorem, ideal gas, Gibbs paradox
3. The Gibbs Distribution
    3.1 Canonical and Grand Canonical ensembles.
    3.2 Minimum free energy principle.
    3.3 Fluctuations in equilibrium.
4. Quantum Statistical Mechanics
    4.1 Postulates and foundations, density matrix, canonical and grand canonical ensembles.
    4.2 The origin of the Third Law of thermodynamics.
5. Ideal quantum and classical gases
    5.1 The Maxwell-Boltzmann distribution.
    5.2 Monatomic and polyatomic classical gasses.
    5.3 Fermi gas.
    5.4 Bose gas; photons, black body radiation, phonons, …
    5.5 Bose-Einstein condensation in an ensemble of a fixed number of non-interacting particles.
6. Phase equilibrium
    6.1 Phase equilibrium
    6.1 Critical point
    6.2 Van der Waals equation of state.

6.3 Equilibrium among different species, e.g., in chemical reactions.

## Electricity and Magnetism

1. Electrostatics
   1.1 Gauss' law
   1.2 Potential.
   1.3 Poisson's equation.
   1.4 Surface charge distributions.
   1.5 Green's function with boundary values.
   1.6 Capacitance, dielectrics..
2. Techniques of solutions of boundary value problems: images, separation of variables, orthogonal functions, Fourier series and integrals, spherical coordinates, spherical harmonics, Legendre functions, Bessel functions, Green's function in spherical coordinates, multipole expansions.
3. Magnetostatics
   3.1 Vector potential
   3.2 Magnetic moment, torque.
   3.3 Permeability
   3.4 H and B.
   3.5 Boundary value problems.
4. Faraday's law
   4.1 Induction.
   4.2 Energy in a magnetic field.
   4.3 Quasistatic fields.
5. Maxwell's equations
   5.1 Gauge choices
   5.2 Retarded solutions.
   5.3 Poynting's theorem.
6. Lorentz transformation of fields, 4-tensor notation for Maxwell's equations.
7. Plane E.M. waves, polarization, plane interfaces.
8. Dispersion of waves, Kramers-Kroning relation, causality.
9. Waveguides, TEM, TE, TM waves.
10. Spherical waves, radiating systems, multipole expansions, Rayleigh scattering.

## Mathematical Methods

1. Linear algebra
   1.1 Finite-dimensional vector spaces, liner maps (operators).
   1.2 Matrices as linear maps
   1.3 Kernal and image sub spaces.
   1.4 Determinants.
   1.5 Inverse maps/matrices
   1.6 Eigenvalues and eigenvectors, characteristic equation.
   1.7 Jordan normal form..
   1.8 Bilinear forms.
   1.9 Orthogonal and unitary maps/matrices
   1.10 Schwartz inequality.
   1.11 Gram-Schmidt orthogonalization.
   1.12 Diagonalization of Hermitian and unitary maps.
   1.13 Tensor algebra.
2. Group theory
   2.1 Homomorphism of groups, isomorphism.
   2.2 Subgroups, normal subgroups, kernel and image of a homomorphism.
   2.3 Introduction to Lie groups and Lie algebras.
   2.4 Linear representation of a group.
   2.5 Irreducible representations.
   2.6 Direct sums, full reducibility.
   2.7 Application to symmetries (in, e.g., quantum mechanics).
3. Real analysis
   3.1 Sequences and series.
   3.2 Convergence, basic convergence tests, absolute and conditional convergence.
   3.3 Alternating series, Taylor series.
   3.4 Asymptotic expansions, asymptotic approximations to integrals.
   3.5 Laplace's method (saddle point).
   3.6 Gaussian integrals.
   3.7 Gamma function.
4. Linear differential equations
   4.1 Infinite-dimensional vector spaces of functions, linear maps (operators).
   4.2 Homogeneous ordinary differential equations (ODE), existence and linear independence of solutions.
   4.3 Wronskian
   4.4 Solution techniques
   4.5 Classification of points (ordinary, regular singular, irregular singular points).
   4.6 Indicial equation and Frobenius method for power series solutions near a regular singular point of a second order ODE.
   4.7 Asymptotic solutions near an irregular singular point, e.g., Bessel's equation.
   4.8 Inhomogeneous equations, Green's function as an inverse operator to a differential operator.
5. Functional analysis
   5.1 Square-integral functions.

5.2 Adjoint operator, role of boundary conditions, Hermitian and self-adjoint operators.
5.3 Eigenvalues and eigenfunctions, spectrum of an operator, spectral theorem.
5.4 Sturm-Liouville theory.
5.5 Examples: Legendre and Bessel functions, orthogonal polynomials.
5.6 Eigenfunction expansion of Green's function.
5.7 Integral transforms: Fourier, Laplace

6. Complex analysis
   6.1 Complex functions, complex differentiability, holomorphic functions.
   6.2 Cauchy-Riemann equations, analytic complex functions.
   6.3 Contour intigrationsn Cauchy's theorem.
   6.4 Singularities: isolated, removable, poles, essential.
   6.5 Analytic continuation, branch cuts, multivalued "functions".
   6.6 Calculus of residues.
   6.7 Green's functions, retarded response, Kramers-Kronig relations.
   6.8 Steepest descent method (in the complex plane)

## Classical Mechanics

1. Elementary principles
   1.1 Mechanics of a particle.
   1.2 Mechanics of a system of particles.
   1.3 D'Alembert principle and Lagrange equations.
   1.4 Conservation theorem.
2. Variational principles
   2.1 Hamilton's principle.
   2.2 Calculus of variations.
3. The central force problem
   3.1 Two body problem and equivalent one body problem.
   3.2 The Kepler problem and planetary motion.
   3.3 Conserved quantities in the Kepler problem: the Laplace-Runge-Lenz vector
   3.4 Scattering by a central force.
4. Rigid body motion
   4.1 Rotations and orthogonal transformations.
   4.2 Euler angles and rotation matrices.
   4.3 Mechanics of rotating systems. Coriolis effect and satellite motion.
   4.4 Euler's equations of motion.
   4.5 The heavy symmetrical top and gyroscopes.
   4.6 Inertia tensor, precession and wobbling of the Earth.
5. Oscillations

5.1 The eigenvalue equations.
5.2 Normal modes.
5.3 Application to vibrations of molecules.
5.4 Forced vibrations.

6. Special theory of relativity
   6.1 Basic postulates.
   6.2 Lorentz transformations.
   6.3 The addition formula and Thomas precession.
   6.4 Covariant formulation.
   6.5 Relativistic collisions.
   6.6 Relativistic forces.
7. Lagrange formulation
8. Hamilton formulation
   8.1 Hamilton's equations.
   8.2 Hamilton's formulation of relativistic mechanics.
   8.3 Principle of least action
9. Canonical transformations
   9.1 Canonical transformations.
   9.2 Poisson brackets and relation to commutators of quantum mechanics.
10. Hamilton-Jacobi theory
   10.1 Hamilton-Jacobi equations.
   10.2 Action-angle variables.