Wormholes and Entanglement

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We will talk about some curious spacetime configurations.

Outline

- Wormholes
  - Science fiction wormholes.
  - Simple wormholes in general relativity
  - Traversable wormhole solutions.
  - A wormhole you could travel through?

- Black holes and quantum systems

- Wormholes and entanglement.

General relativity

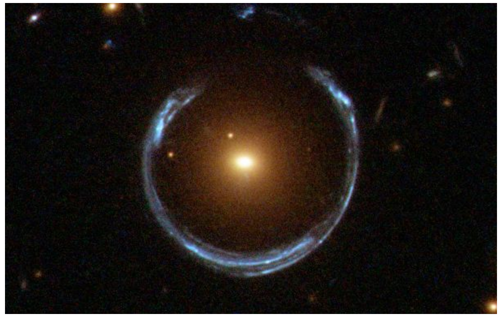
- Einstein realized that gravity can be described by allowing spacetime to be curved.

- Einstein equations:

Curvature = energy density

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$$

- Light rays are typically focused as they propagate on a curved spacetime.

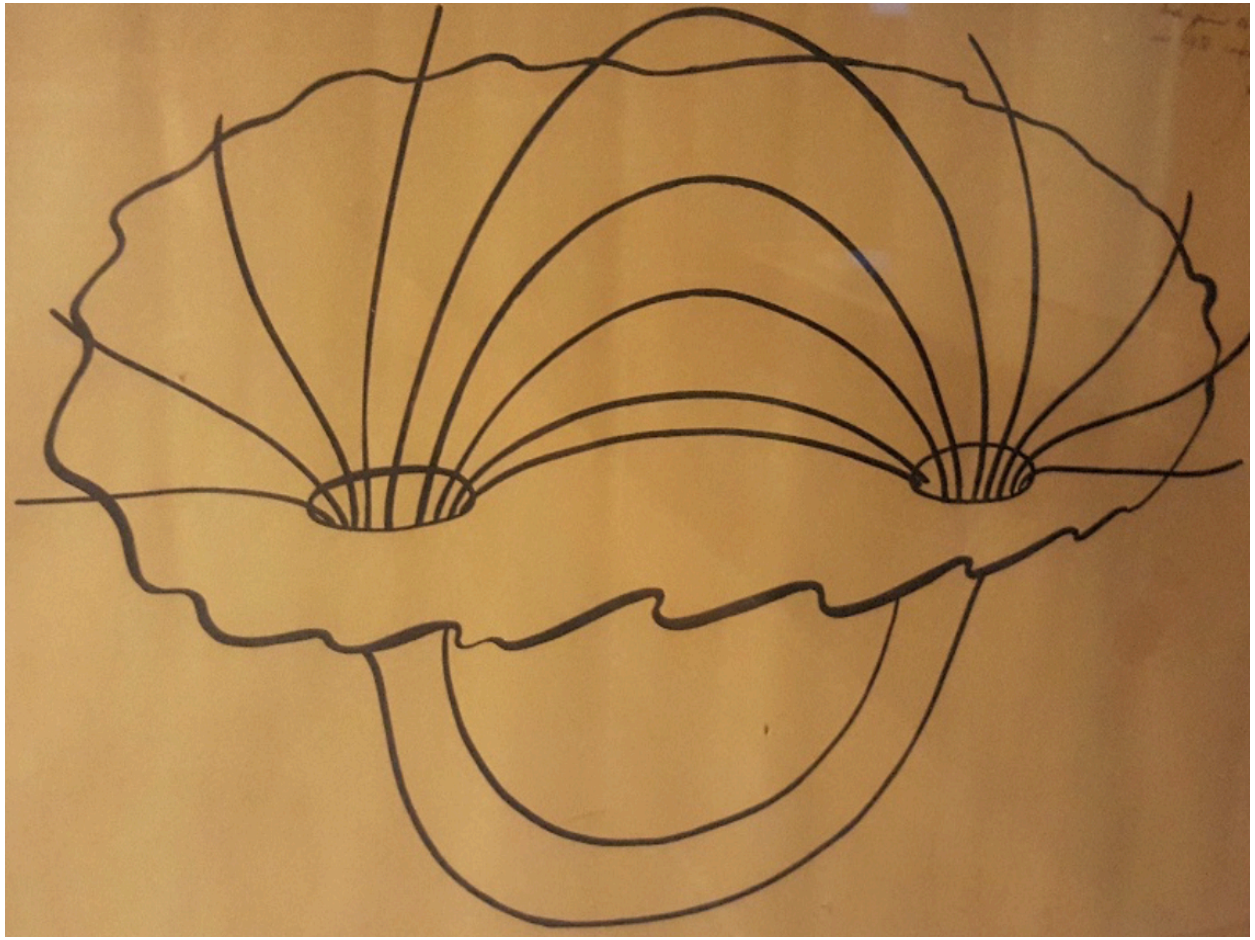


General relativity and spacetime geometry

- General relativity is based on the idea that spacetime is curved.
- In many cases typically considered the deformation of spacetime is small and its topology is the same as that of Minkowski space.

- Here we will analyze spacetimes with non-trivial spacetime topologies.
  - We will first see what is allowed and what is forbidden.
  - Traversable and non-traversable wormholes.
  - Relation to black holes and entanglement.

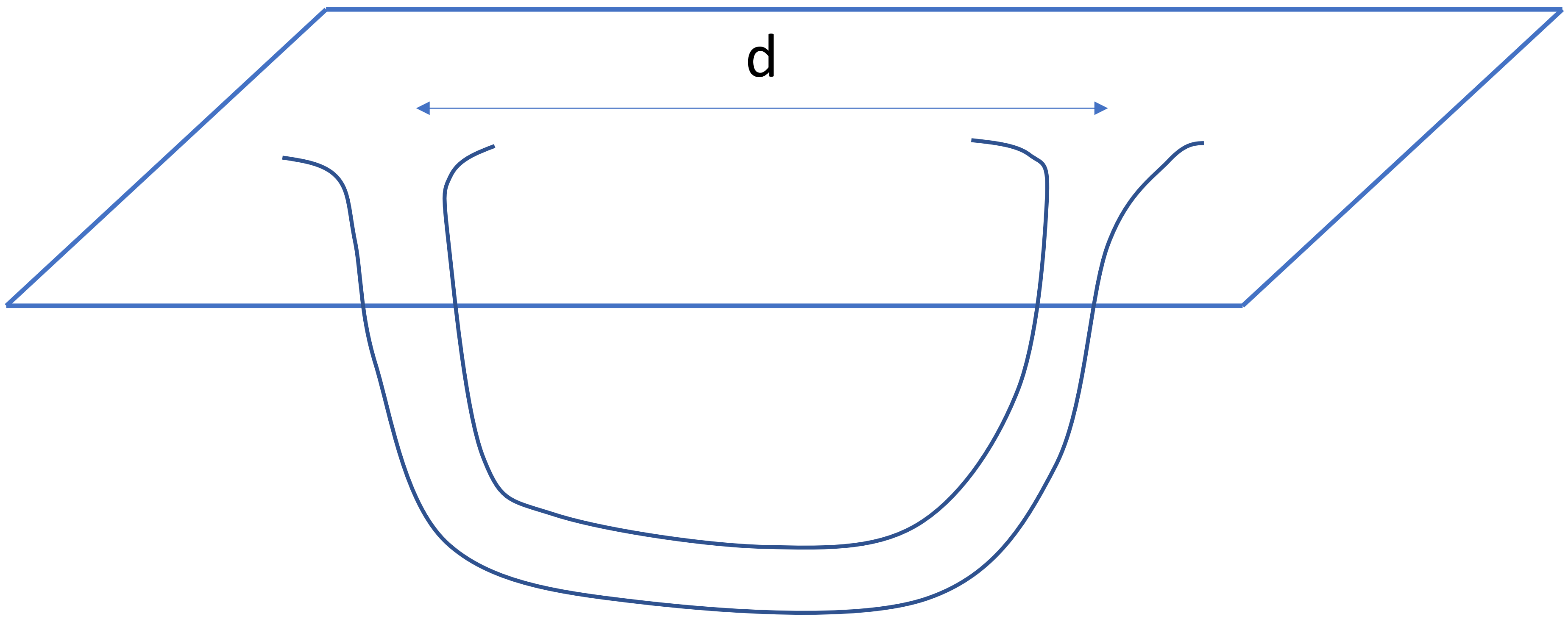
Wormhole = spacetime tunnel



Wheeler 1966

Science fiction wormholes

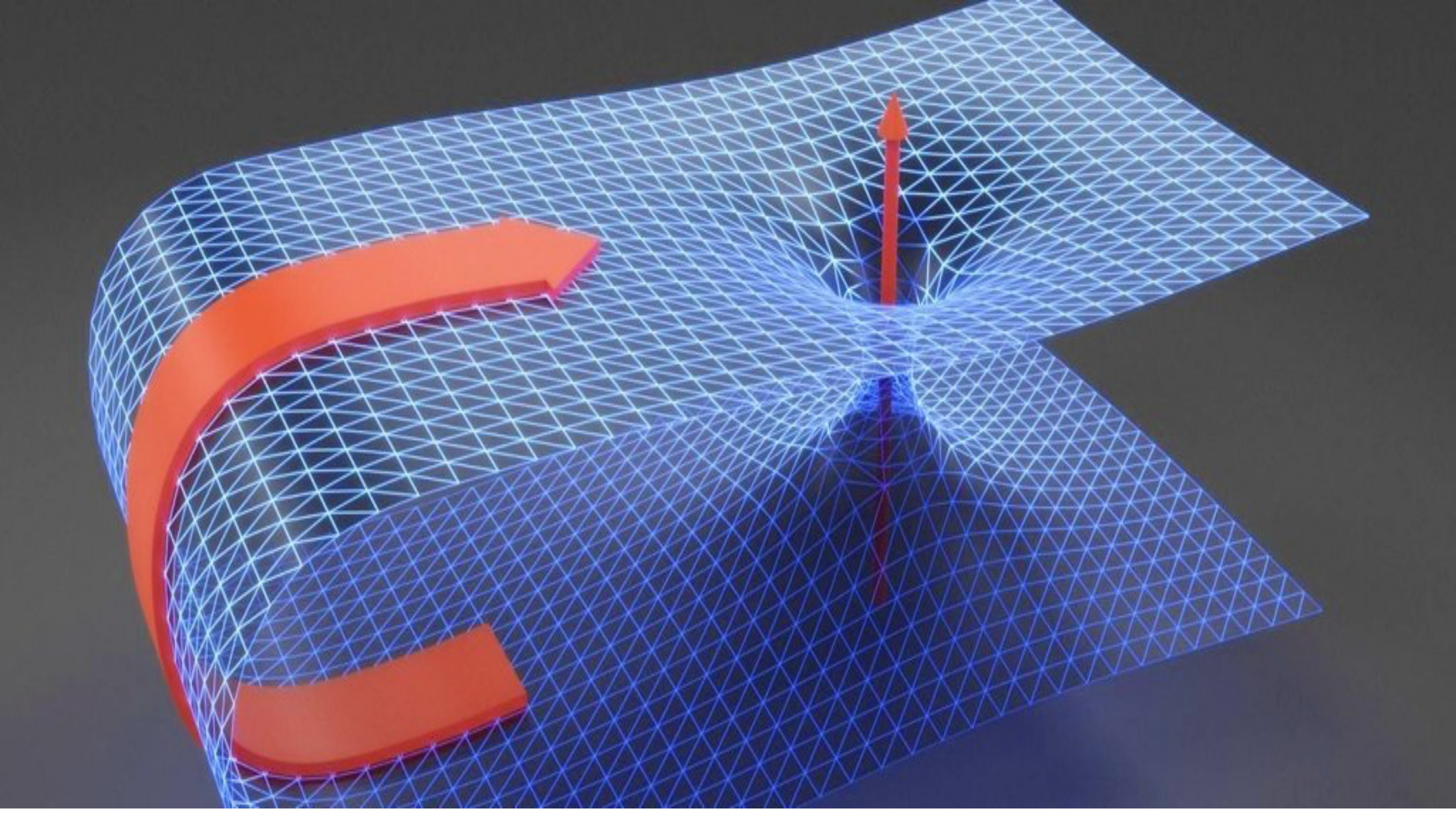
(used to travel in time or faster than light)



Science fiction wormhole: travel time is shorter than d. → would allow faster than light travel, or travel to the past.

(speed of light =1)

(Travel time as seen by somebody that remains outside)



Science fiction wormholes are problematic

- They would violate the principle that signals cannot propagate faster than light in the ambient space.
- We would have a violation of the principle on which special relativity rests.
- This is a basic principle of our current physics framework!.
- It is one principle that lead to general relativity itself.
- Combined with special relativity, it can lead to travel into the past, with the associated paradoxes.

- But they are certainly possible as mathematical geometries…

Einstein's equations to the rescue

- The geometry should obey Einstein equations.

Curvature = energy density

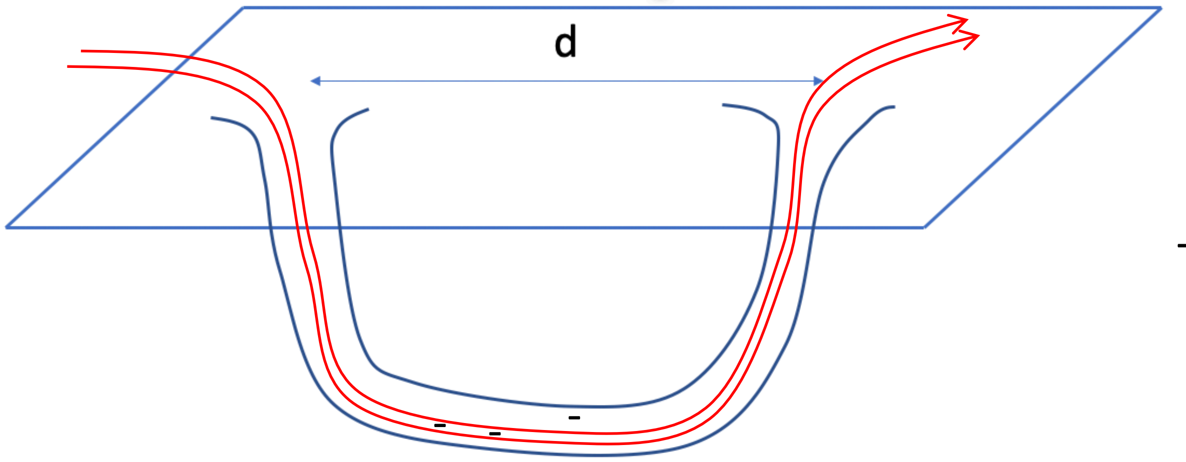
- Could we engineer the appropriate matter distribution?
- But energy is positive (classically) ($T_{++} \geq 0$)
- This results in the condition that gravity always <u>focuses</u> light rays.

Side comment: Focusing of light and black holes

- The fact that matter focuses light is central to one of Roger Penrose's most famous results, which is that black holes can form and that they contain a singularity.

Traversable wormholes need defocusing

They focus as they go into one mouth



They defocus as they go out.

→ There must be negative null energy in the wormhole

- This seems a happy resolution. Einstein’s equations do not allow science fiction wormholes!

- Einstein theory is more than geometry. The geometry is constrained by the equations. And these constraints depend on the properties of matter.

- It is a delicate dance between geometry and matter.

But in quantum mechanics the energy can be negative!

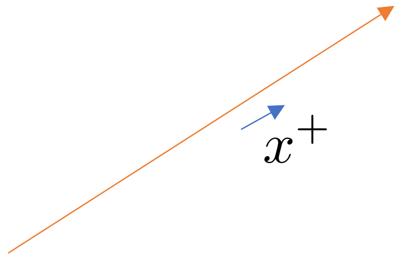
Casimir forces: $T_{++} < 0$ in some regions.

Can this allow wormholes ?

There is an integrated null energy condition

- Achronal, average null energy condition:

$$\int_{-\infty}^{\infty} dx^+ T_{++} \geq 0$$



- If you have a complete fastest light ray in a given geometry, then the integral of T_{++} should be positive (or zero).
- You can have negative T_{++} in some regions, but the total integral should be positive.

This is a conjecture in general. In flat space it was proven for general interacting theories.

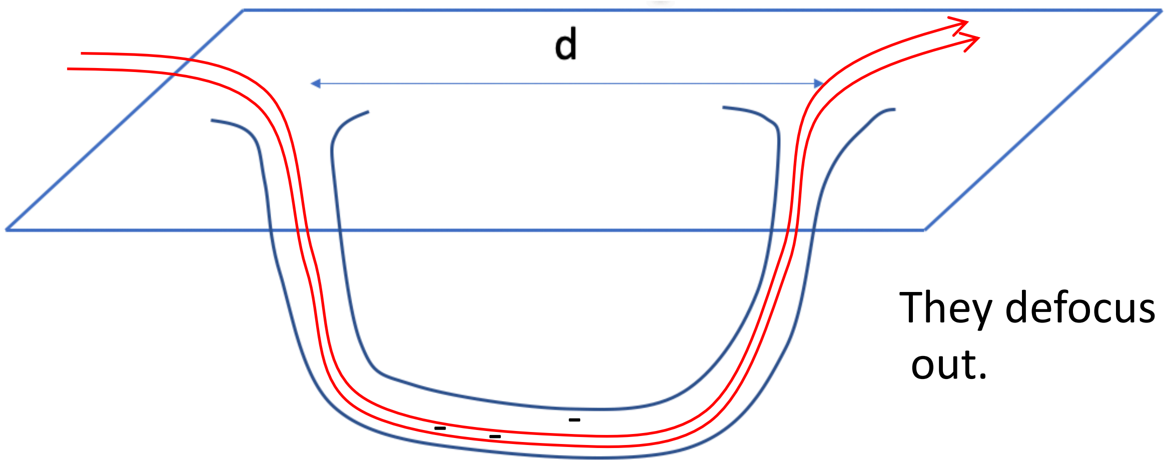
Faulkner, Leigh, Parrikar, Wang; Hartman, Kudu, Tajdini

Fuller, Wheeler, Friedman, Schleich, Witt, Galloway, Wooglar

Science fiction wormholes are forbidden

- If the wormhole allows you to go faster than light in the ambient spacetime. Then the light ray going through the wormhole is a "fastest" (achronal) light ray.
- Total focusing is equal to the integral of the null energy.
- So they cannot defocus….

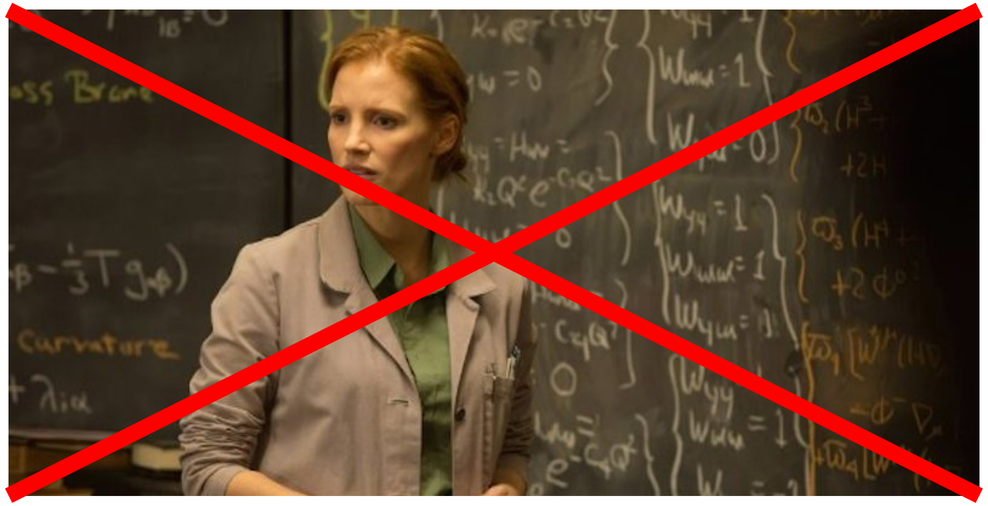
They focus as they go into one mouth



They defocus as they go out.

Conclusion

- Basic principles of special relativity and quantum mechanics imply positive energy.
- Together with general relativity, these forbid traversable wormholes violating the faster than light travel ban. (not a rigorous theorem yet)
- These are very basic principles, not the specific properties to the particles know. Dark matter is also expected to obey these properties.



Comment

- An interesting link between causality and a certain positivity property of property of quantum matter.

- Gravity and quantum mechanics are good partners.

- The beast saving the beauty.

- ``Exotic matter'' → matter outside the framework of Quantum Field Theory.

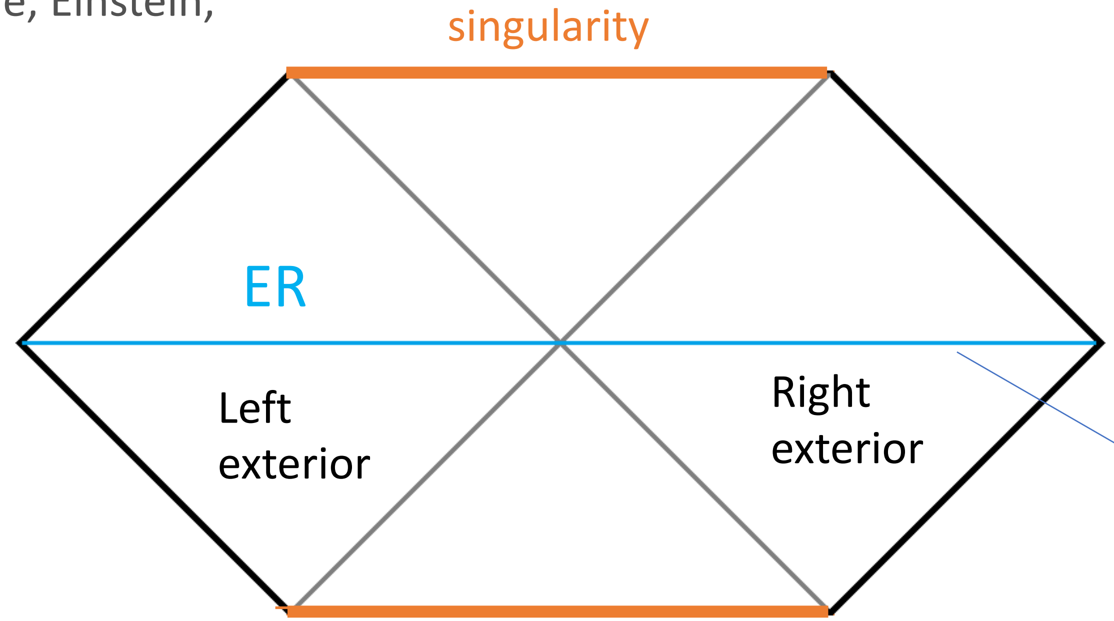
But the laws of physics allow non-traversable wormholes

These are time dependent wormholes.

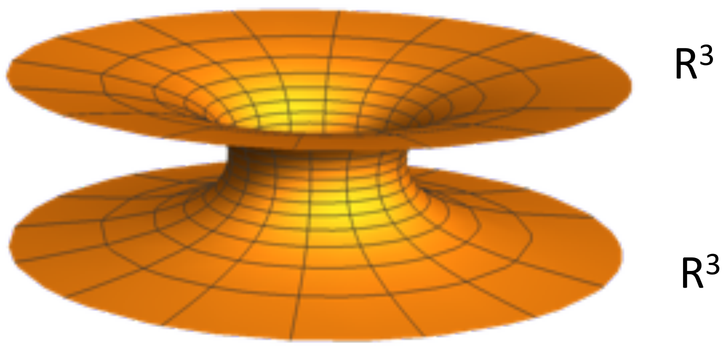
In fact, these arise in the simplest solution of general relativity: the Schwarzschild solution.

Full Schwarzschild solution

Eddington, Lemaitre, Einstein,
Rosen, Finkelstein,
Kruskal



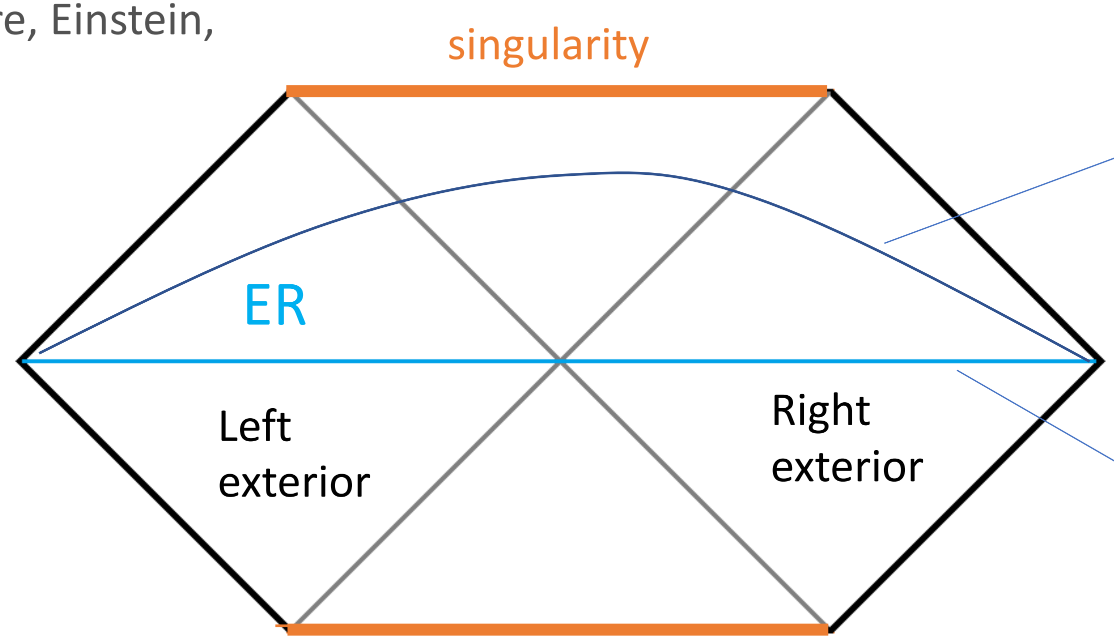
Einstein-Rosen bridge

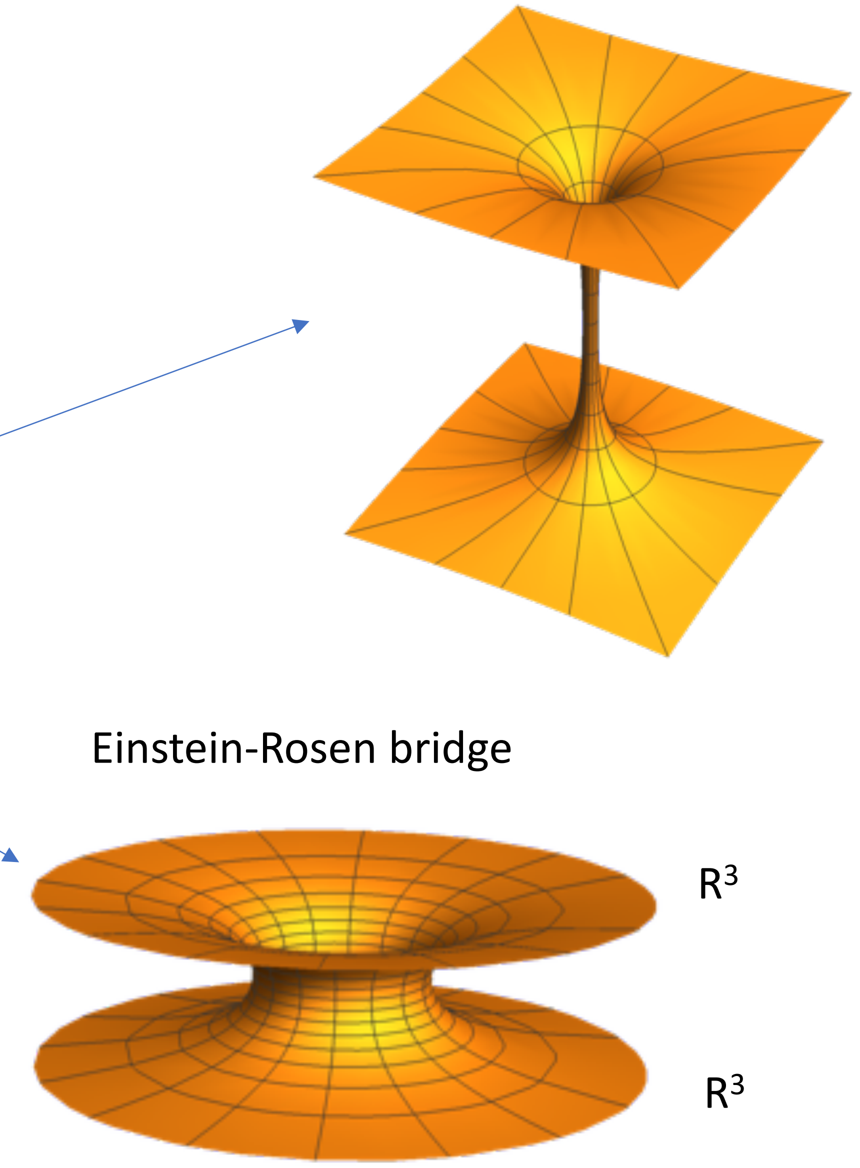


Vacuum solution.   No exotic matter.
Two exteriors, sharing the interior.

A collapsing bridge

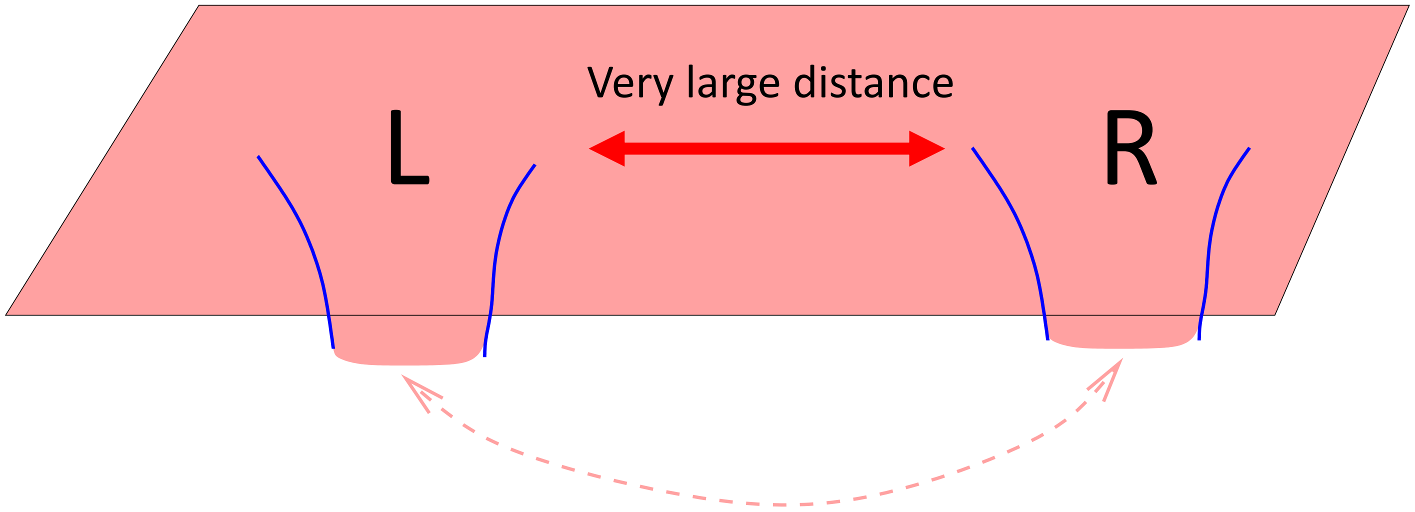
Eddington, Lemaitre, Einstein,
Rosen, Finkelstein,
Kruskal

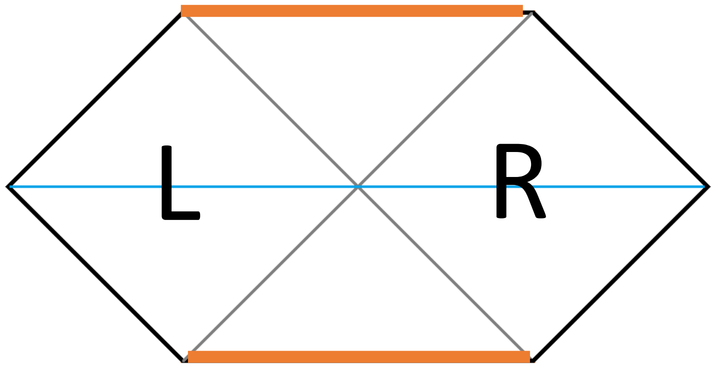




Vacuum solution. No exotic matter.
Two exteriors, sharing the interior.

Wormhole interpretation, similar solution.





Non traversable

No signals

No causality violation

What are we to make of these geometries?

- 1) Ignore them. There is no simple formation mechanism. If we had two separately collapsing black holes, there is matter inside and the geometry would not be this.

- 2) Try to understand what them better. We will come back to them later.

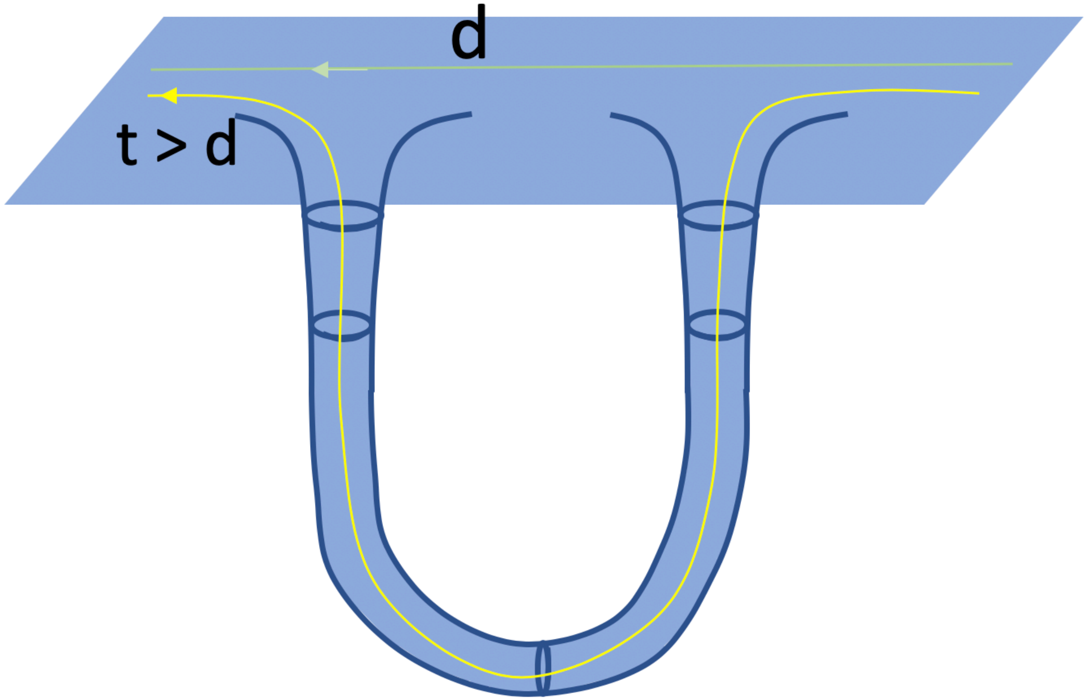
Are traversable wormholes possible?

- No, if they allow you to travel faster than light (science fiction wormholes).

- Yes, if the travel time through the wormhole is longer than the travel time outside the wormhole.

Not shortcuts, but longer detours ?

- There can be wormholes where the observer going through the wormhole takes longer than one going outside the wormhole.



- This requires quantum effects. They are forbidden classically.

We will discuss now these wormholes

They are traversable. 🙂
They have no horizon.
They do NOT allow faster than light travel. ☹️ 🙂

Note: the observer's proper time through the wormhole can be much shorter than the outside travel time!

The constructions we discuss require special matter, but still matter obeying the rules of QFT. No ``exotic'' matter.

Simplest example

It requires some specific matter content…

JM, Milekhin, Popov

The theory

$$S = \int d^4x \left[R - F^2 + i\bar{\psi} \not{D} \psi \right]$$

Einstein + U(1) gauge field + massless charged fermion

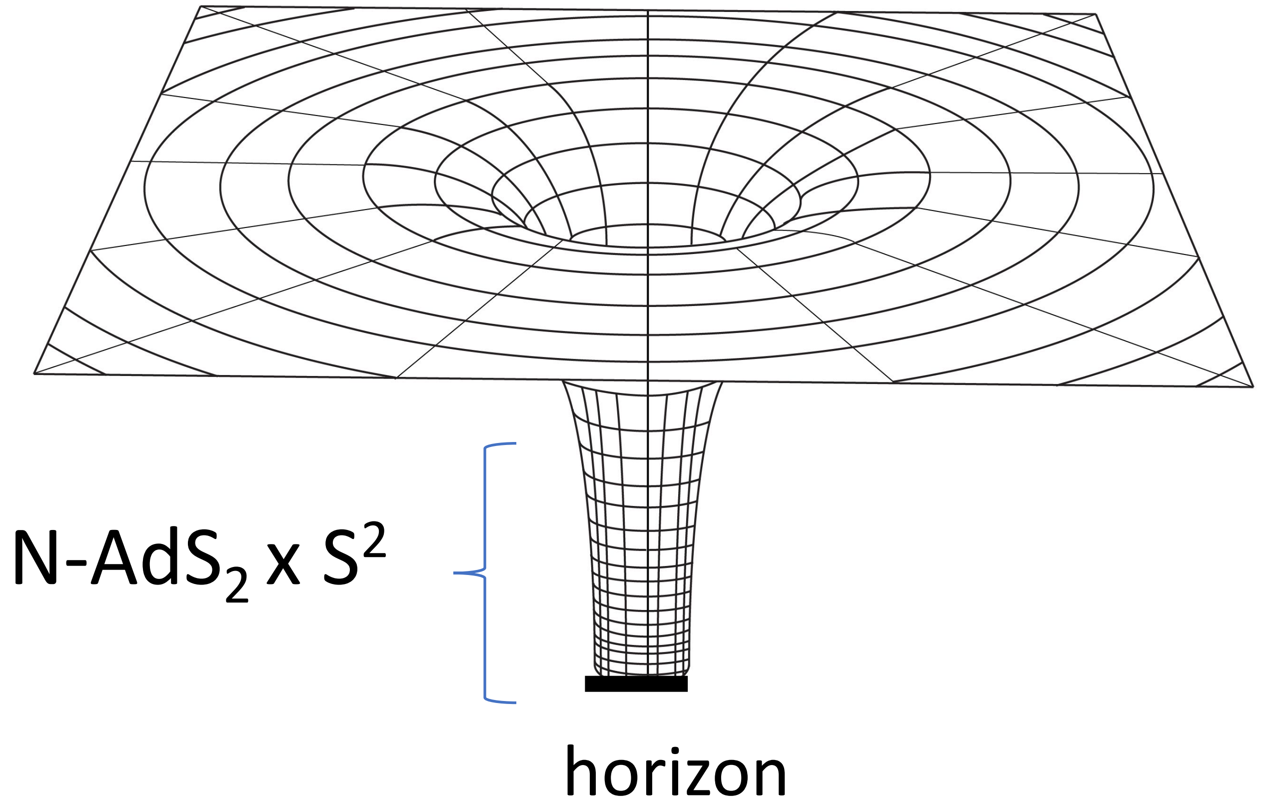
Could be the Standard Model at <u>very small</u> distances, with the fermions effectively massless. The U(1) is the hypercharge. SU(3) x SU(2) x U(1).

$l_{\text{Planck}} = 1$

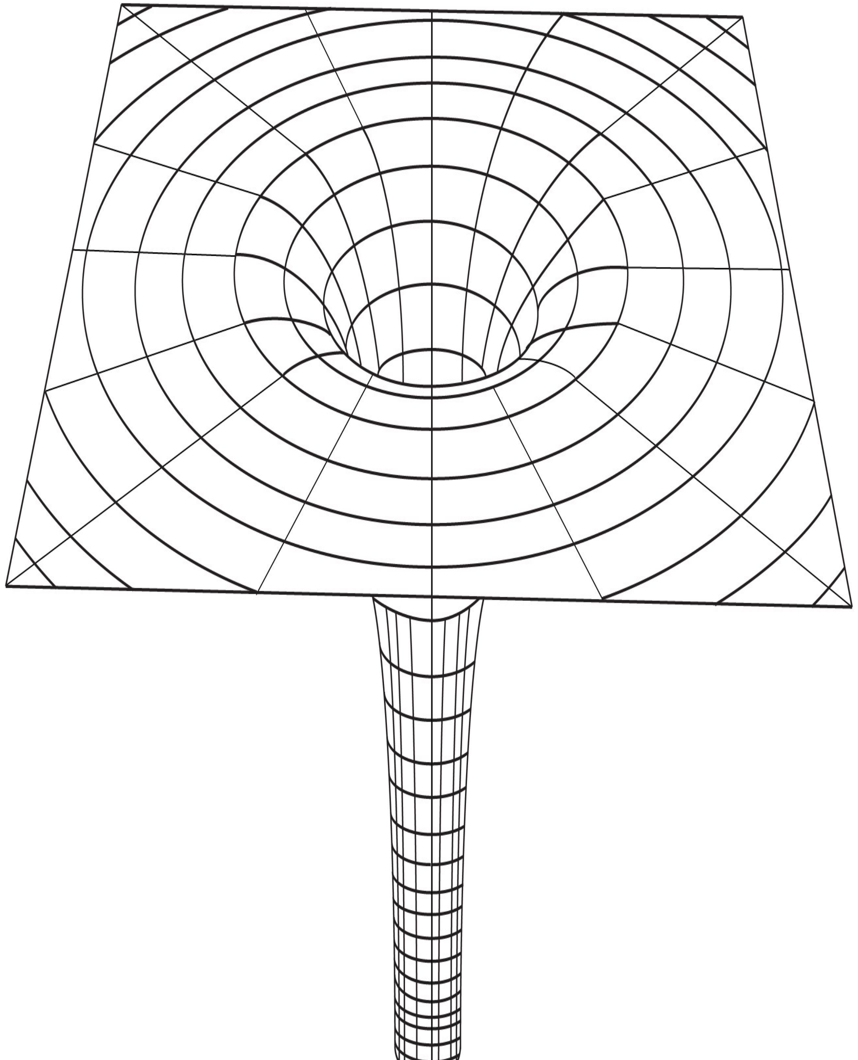
Near extremal magnetically charged black holes

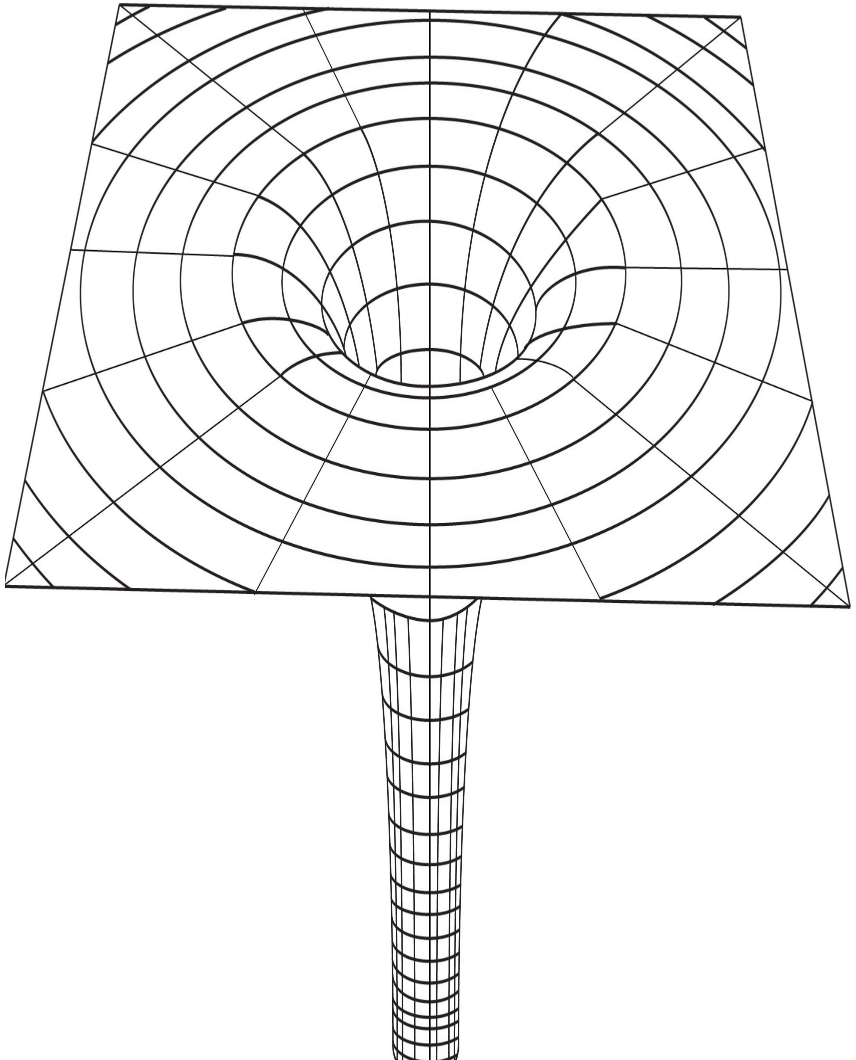
$$M \geq Q$$

$$M \sim Q$$

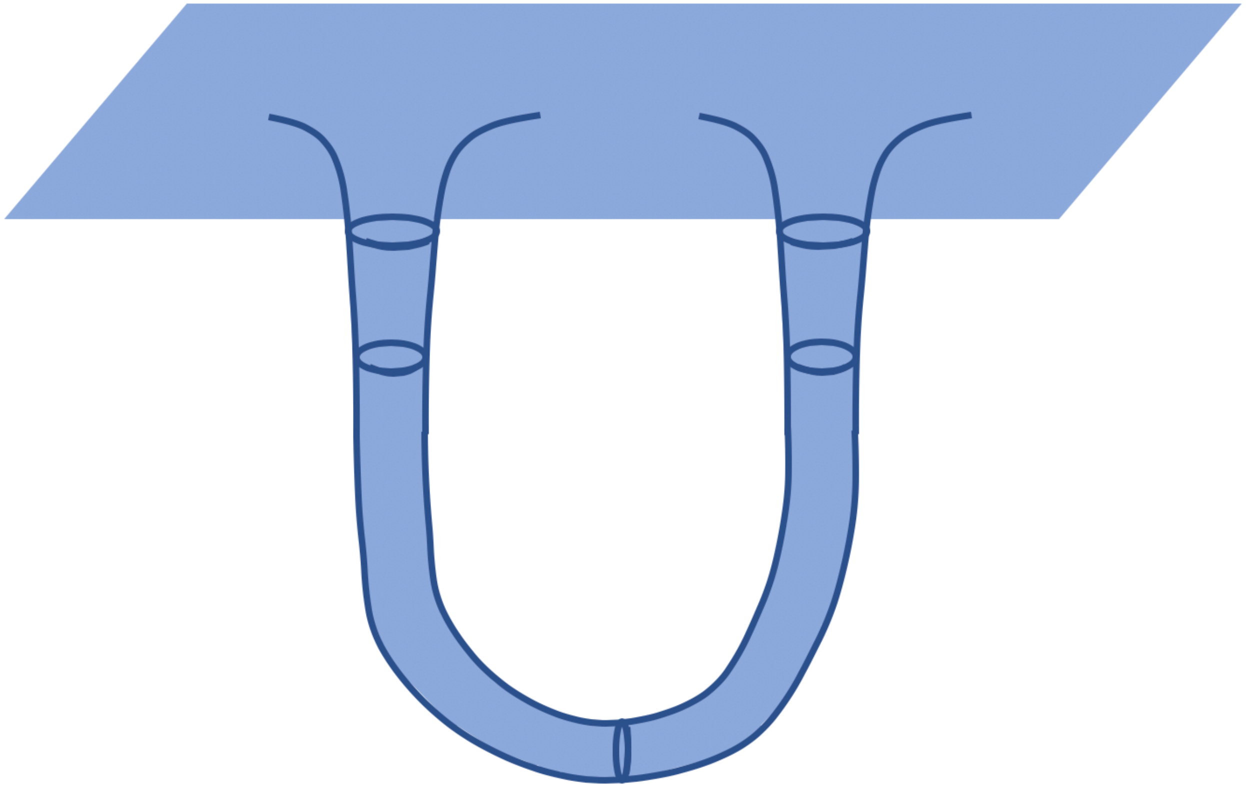


Consider a pair with opposite magnetic charge





Connect their throats



No horizon

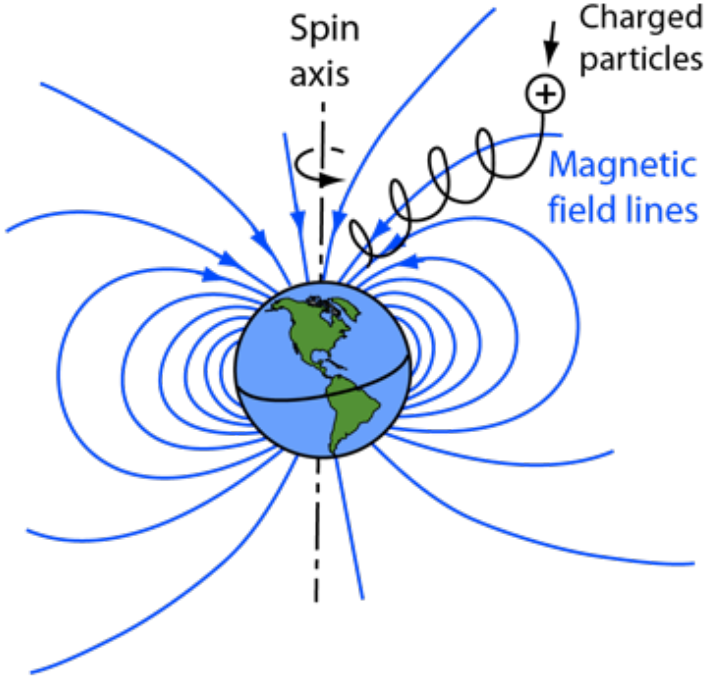
This only requires a ``small'' deformation of the geometry

There is one free parameter at this stage, which is the length of the wormhole.
We will fix it later.

Now let us consider the fermions

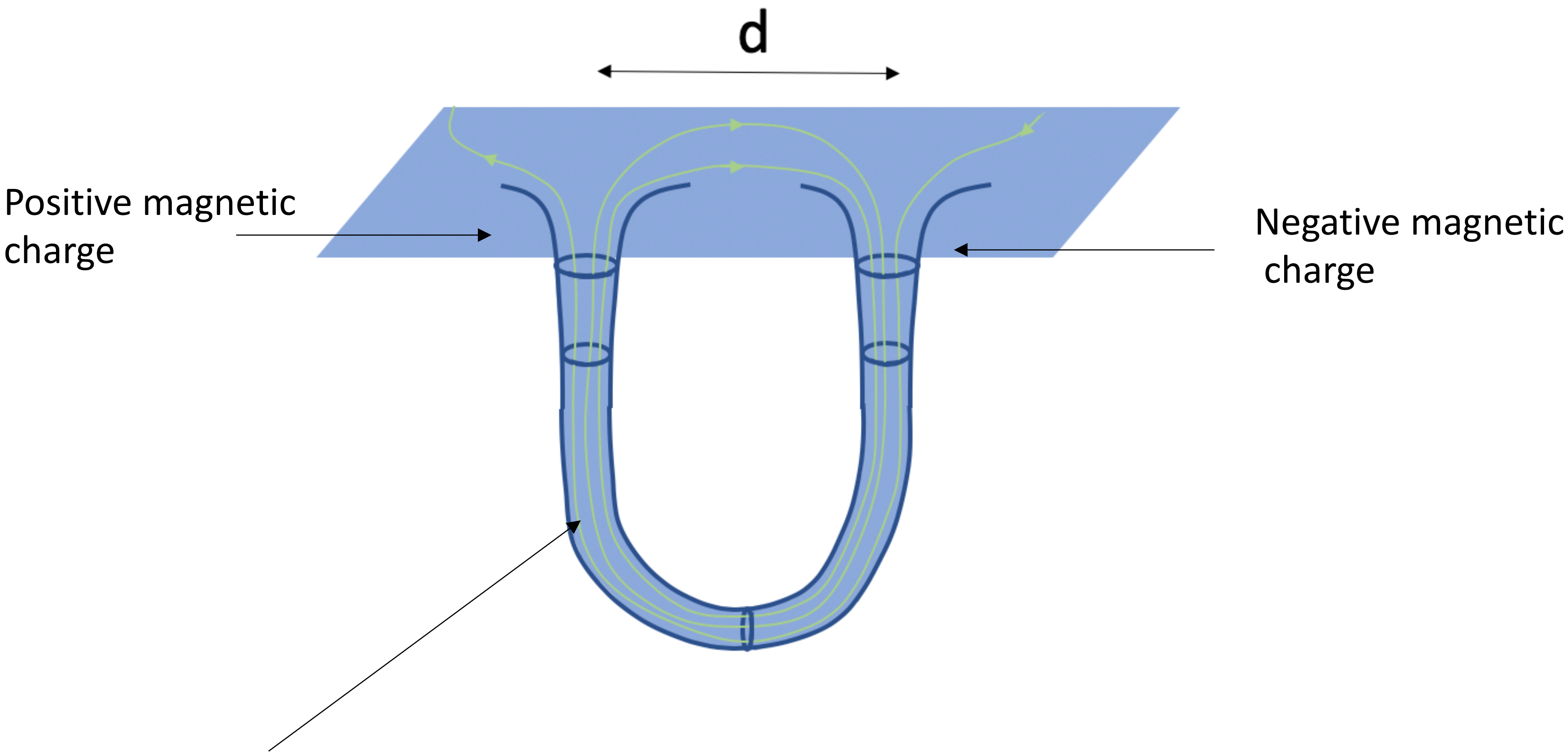
One 4d massless fermion → set of q two dimensional massless fermions along the field lines. (q = integer magnetic charge >> 1)

Similar to:



Lowest landau level → zero energy
(cancellation between orbital and magnetic moment contributions)

Fermion trajectories



Charged fermions moves along these closed circles, following magnetic field lines. One per flux quantum.
q 2d fields in total.

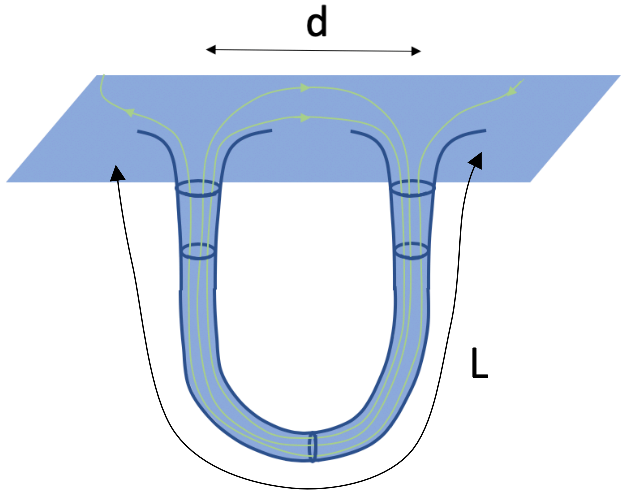
Casimir energy

Assume: “Length of the throat” is larger than the distance. (Here we define L as the time it takes to go through the wormhole as seen from the outside).

Assume L > d.

Casimir energy is of the order of

$$E \propto -\frac{q}{L}$$



- Including this energy in Einstein's equation → Fix the parameter L.

- It is basically an energy minimization problem.

$$E = \overbrace{\frac{r_e^3}{G_N L^2}}^{\text{Energy from bending the throats}} - \overbrace{\frac{q}{L}}^{\text{Casimir energy}} \sim \frac{q^3 l_p}{L^2} - \frac{q}{L}$$

$$\frac{\partial E}{\partial L} = 0 \longrightarrow L \sim q^2 l_p = q r_e$$

This solves the Einstein equations in the wormhole region.

- To prevent the two mouths from falling into each other → make them orbit around each other.

- Gives a long lived state. It will slowly decay be emitting electromagnetic and gravitational radiation.

Time it takes to go through the wormhole ?

- As measured from the observer that remains outside: $L > d$.

- As seen by the observer traveling through: $r_e << L$. Could be much smaller than d.

Production?

- We have only argued that these are possible long lived solutions.

- I do not know of any simple way of producing these configurations if they were not initially present.

Summary

Wormholes

Non-traversable ✓

(simple solutions)

Traversable

Slower than light travel ✓

(solutions incorporating quantum effects)

~~Faster than light travel~~

(not possible within quantum mechanics and relativity)

Now a science fiction exercise.

Could it be ``possible'' for us to travel though wormholes in our universe?

- The ones with Standard Model matter can only be microscopic.

- For macroscopic ones we need extra ``massless'' matter.

- We postulate an extra ``dark sector''. Involving an an extra dimension, a 5th dimension. We have gravity plus a gauge field in the extra dimension.

Randall Sundrum II model (1999)

- We can then get a traversable wormhole similar to the one we discussed that is big enough for us to travel through it.

Some numbers

- Size of the mouths: size of the earth. (so that curvature do not kill us)
- Travel time as seen from the outside: about 20,000 years. (as short as we could make it given the constraints on the size of extra dimensions, $R < 50$ microns)
- Travel time as seen by the traveler, less than a second.
- Many ``practical problems”: CMB entering from the other side would fry us in the middle. We would not have a simple formation mechanism, etc.

With the laws of physics we know, and the present constraints on new types of matter, it seems very hard to make humanly traversable wormholes…

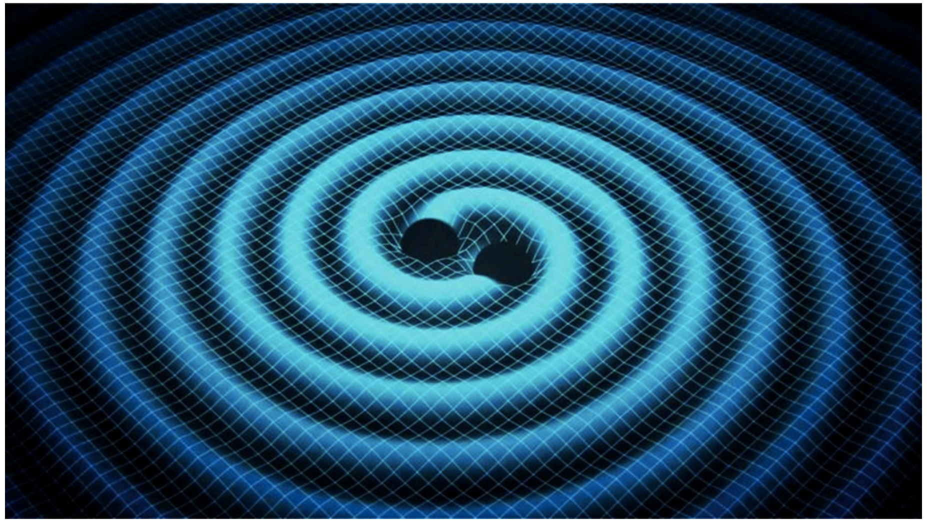
But it is ``in principle'' possible.

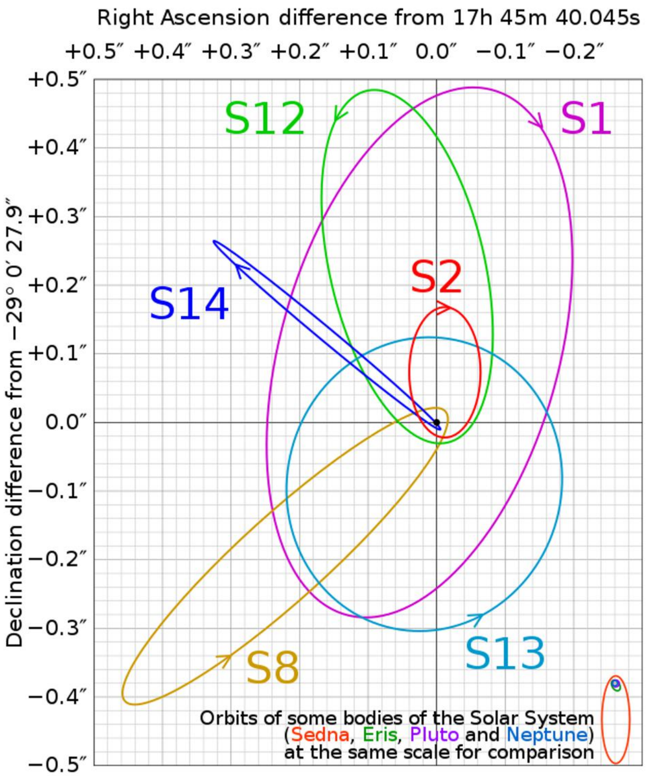
End of the science fiction exercise

But this is not really the reason why I am telling you all of this about wormholes.

The most interesting aspect of all these wormholes is their connection to black holes and entanglement.

Black holes





They have been in the news.

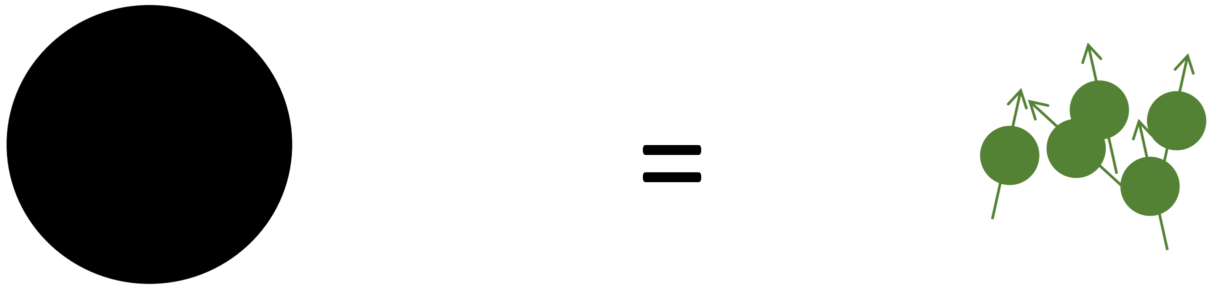
We will be interested in their quantum aspects

- They emit hawking radiation.
- They have an entropy proportional to their area.
- They obey the laws of thermodynamics.

- This has suggested a hypothesis:

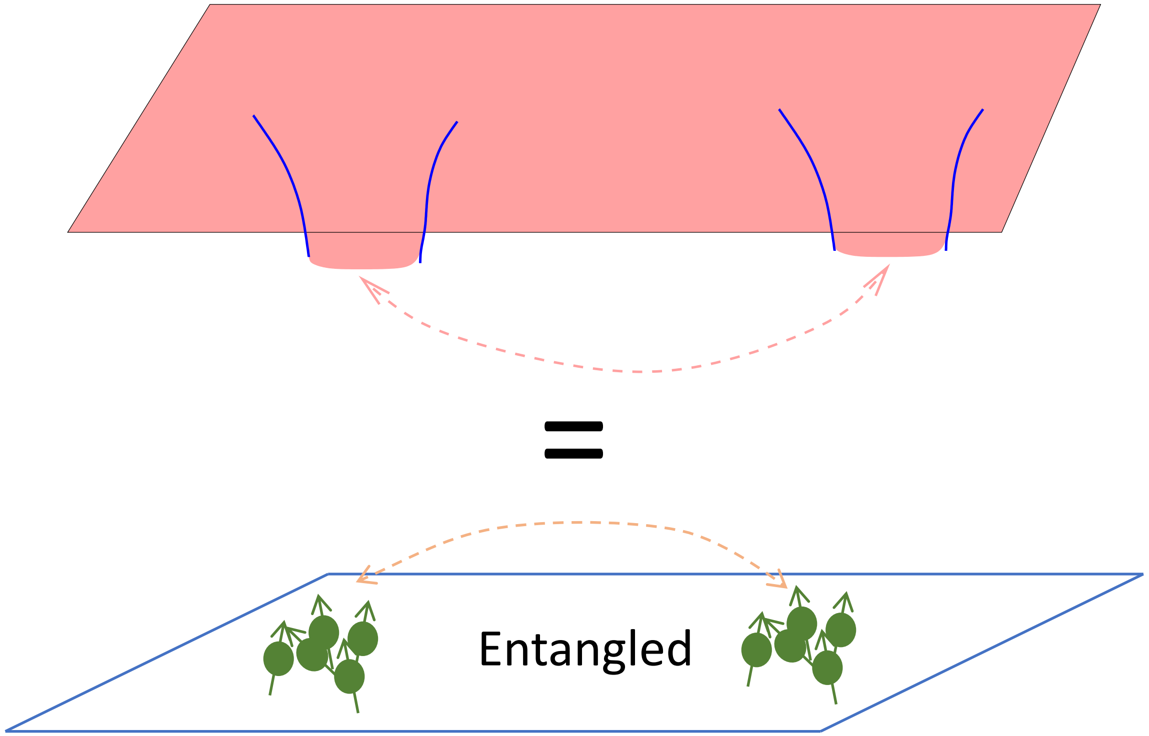
A black hole as a quantum system

- A black hole seen from the outside can be described as a quantum system with S degrees of freedom (qubits). S = Area/4   (l_p =1)
- It evolves according to unitary evolution, as seen from outside.



- This hypothesis is supported by results in string theory.
  - Entropy of supersymmetric black holes — Strominger Vafa, … Sen,…
  - AdS/CFT or holography — JM, Gubser, Klebanov, Polyakov, Witten
  - …

Non traversable wormholes and entangled states



Two black holes connected through
 the interior
=
Two entangled black holes.

W. Israel
JM
JM, Susskind

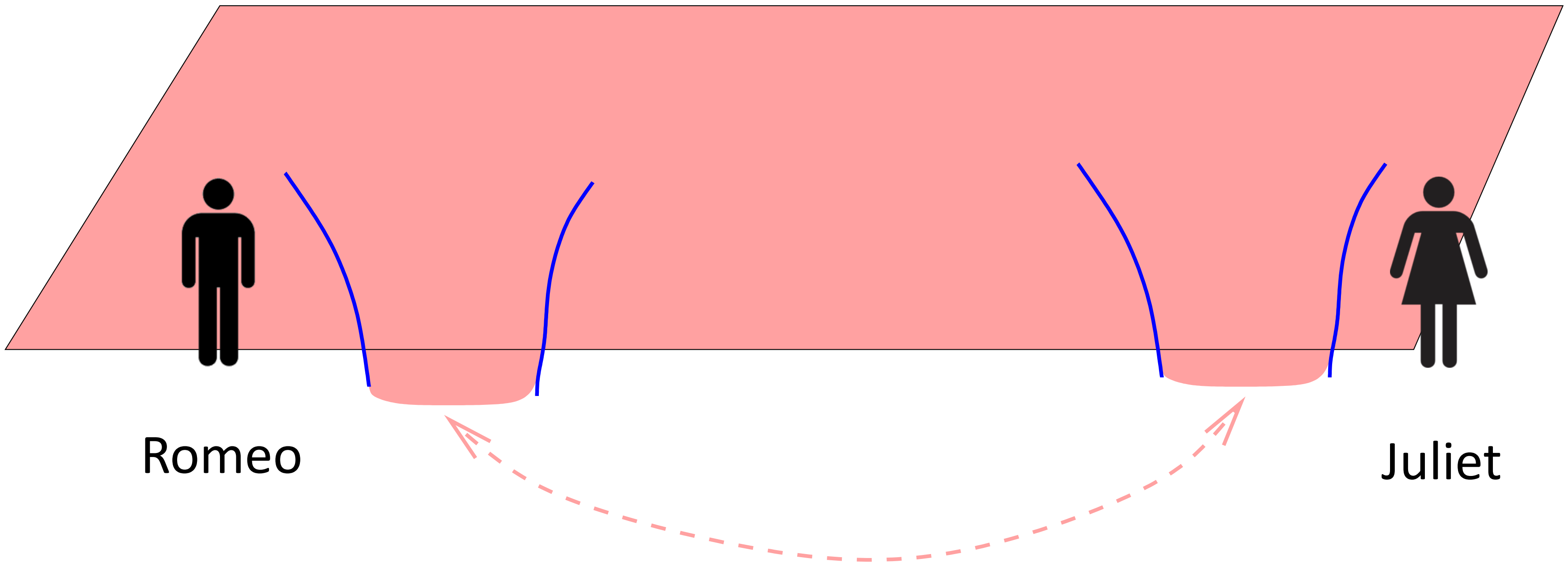
ER = EPR

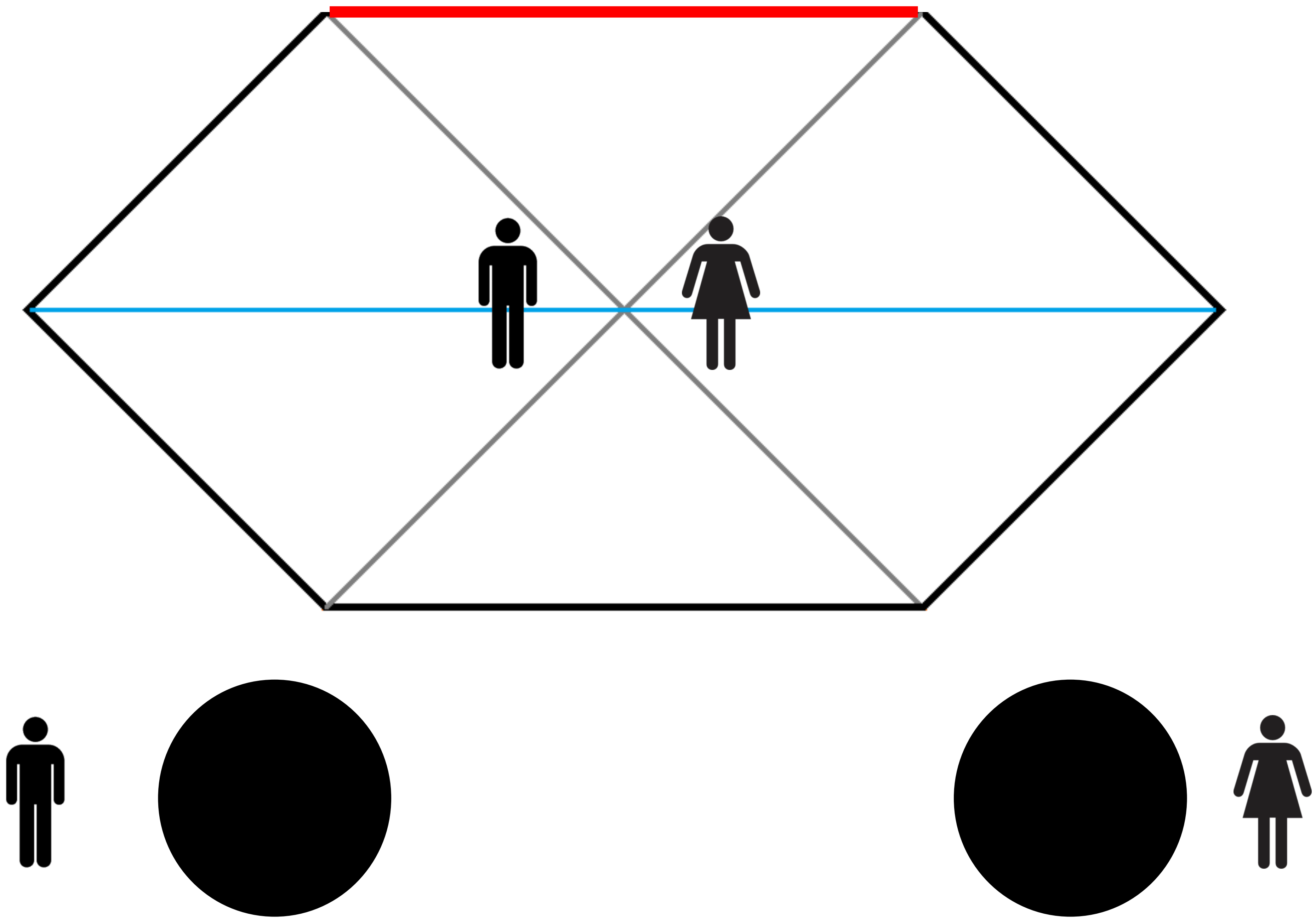
In a particular entangled state

$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

(state at t=0)

A forbidden meeting





Is there a better way ?

- Quantum teleportation
- Traversable wormholes

Gao, Jafferis, Wall,
…

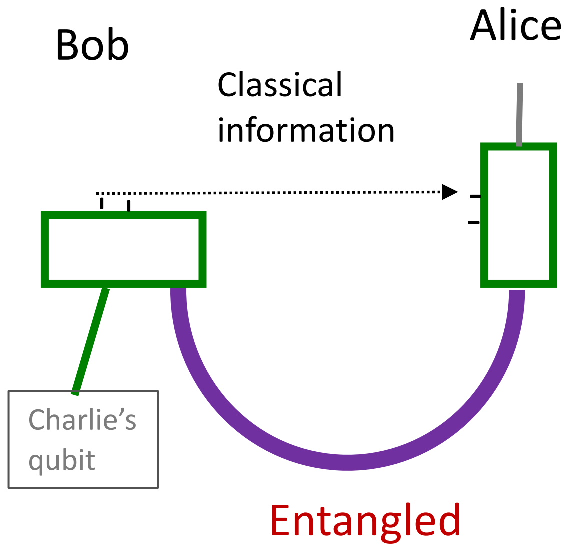
Gao, Jafferis, Wall

Quantum teleportation and traversable wormholes

Bennett, Brassard, Crepeau, Jozsa, Peres, Woetters (1993)

Quantum teleportation

- Bob and Alice share an entangled pair of qubits.
- Charlie gives Bob a qubit and he wants to send it to Alice.
- Bob does a joint measurement of Charlie's qubit and his qubit.
- Sends the the result to Alice as classical information
- Alice does an operation on his qubit that depends on Bob's result.
- Alice gets the qubit.



- Resources needed to send a qubit: One entangled qubit and 2 bits of classical information.

Would you like to be teleported ?

Returning briefly to our story…

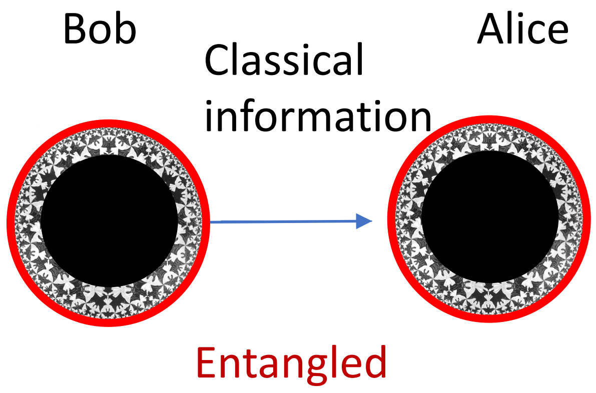
Juliet tries to convince Romeo to build a giant teleportation machine, and suggests he teleports himself.

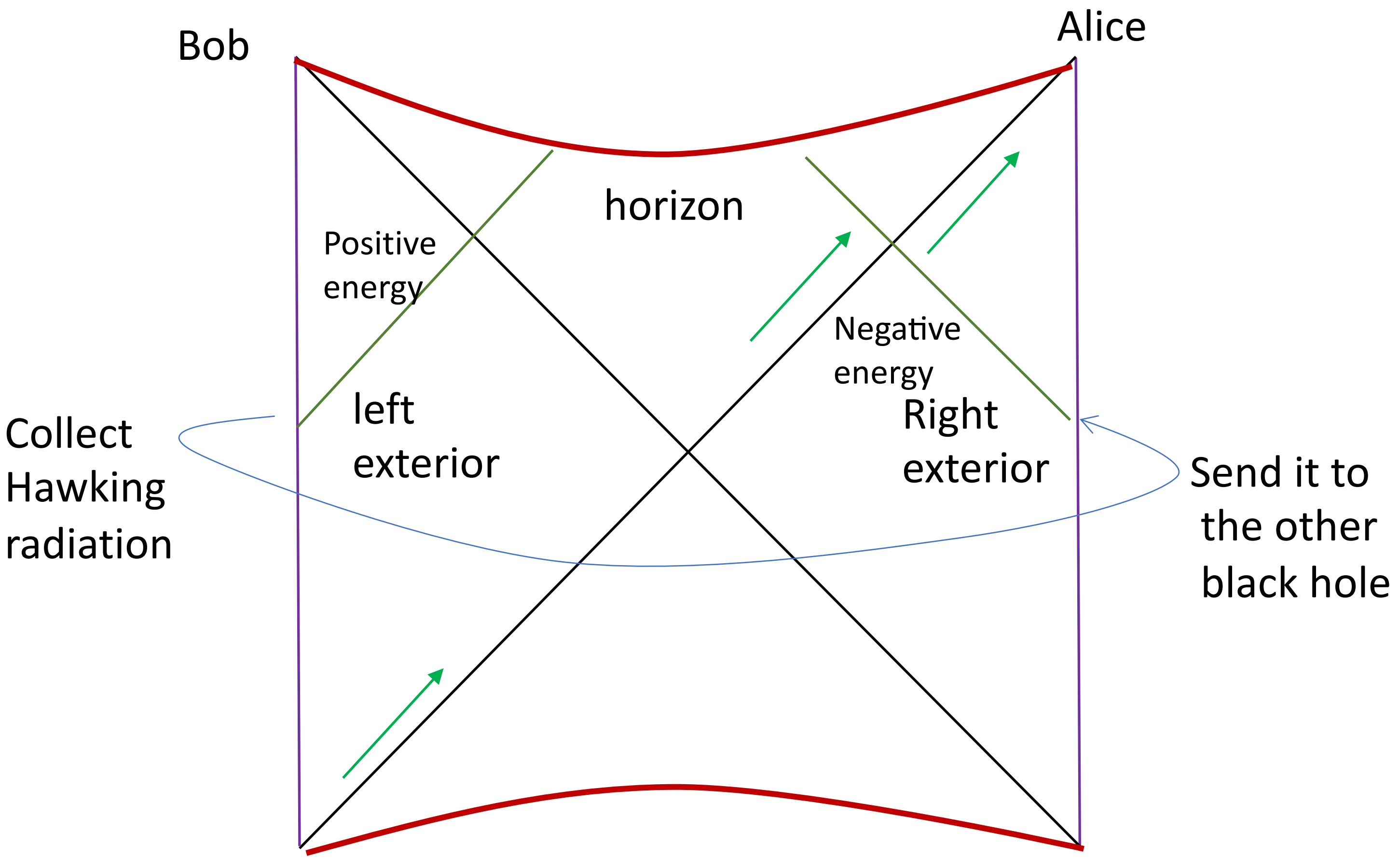
Romeo is worried about the process.

Juliet suggests using entangled black holes

Gao, Jafferis, Wall

Teleportation through the wormhole





Romeo jumps in and meets Juliet.

He reports that he didn’t feel anything strange in the process. It was like travelling through empty space.

That was just a story, but…

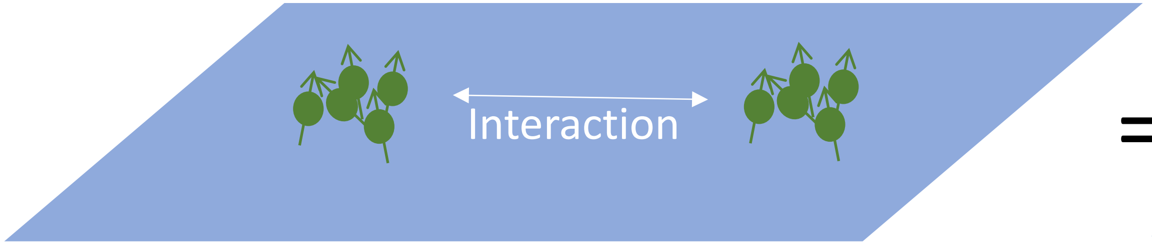
``Quantum gravity'' in the lab

- There are theoretical proposals for doing something analogous to this for somewhat complex quantum systems.
- You would send qubits.
- It is a special type of teleportation.

Schuster, Kobrin, Gao, Cong, Khabiboulline, Linke, Lukin, Monroe, Yosihda, Yao

Nezami, Lin, Brown, Ghharibyan, Leichenauer, Salton, Susskind, Swingle, Walter

Traversable wormholes and entanglement



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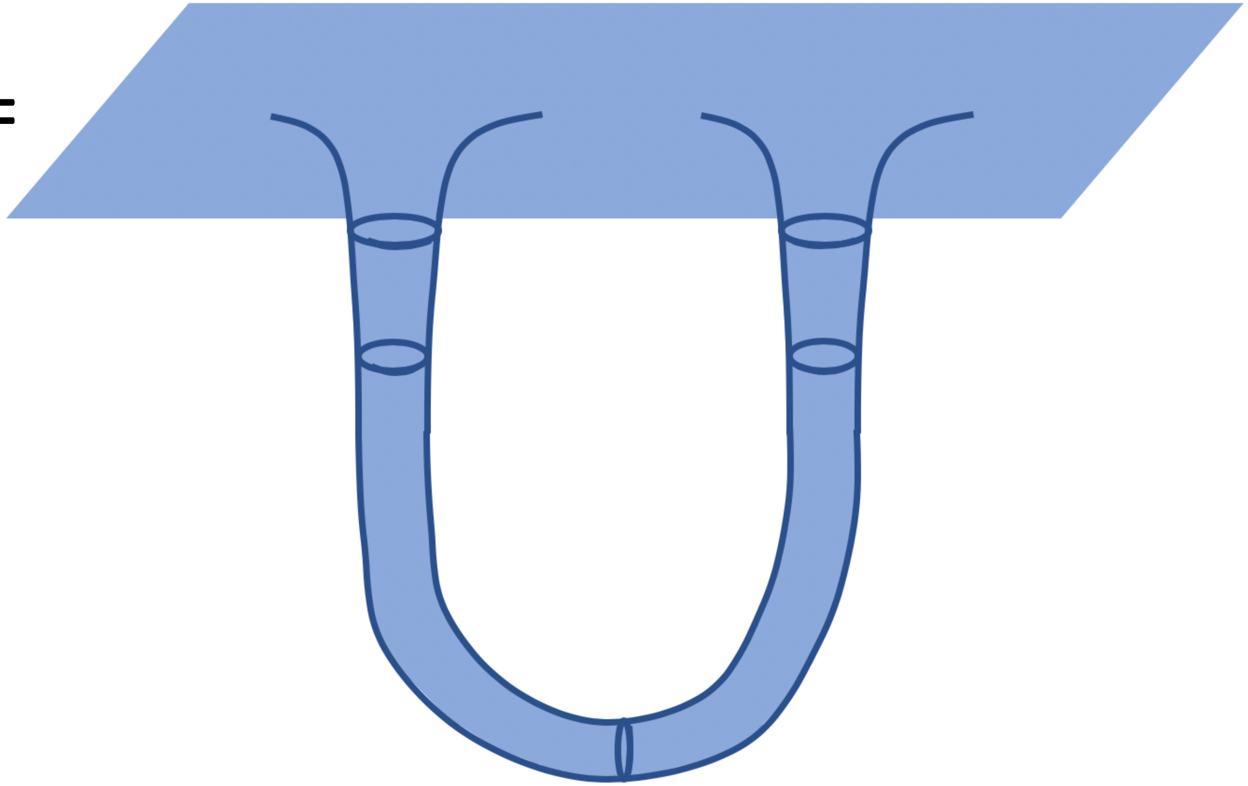

The interaction selects the entangled state as the lowest energy state.

It is somewhat similar to how the Van der Waals interaction produces entanglement among the lowest energy levels of a pair of atoms or molecules.

Conclusions

- Wormholes that would allow faster than light communication, or travel back in time, are forbidden.
- Non-traversable wormholes are simple solutions of general relativity and are closely connected to black holes.
- They can be interpreted as entangled states.
- Entanglement is related to wormholes. ER=EPR.
- When the two black holes interact in the ambient space, they can lead to a traversable wormhole. But one consistent with the faster than light travel ban.

- These ideas are inspiring interesting experiments with manybody systems.

- Similar wormholes play a role in the analysis of the black hole information paradox → We will discuss that in the next lecture.

Thanks!